Критическая динамика развития разрушения и автомодельные закономерности сейсмических событий

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Content

- **Introduction.** **Defects and** mesoscopic states: paradigm in modeling.

- **Defects in condensed matter. Statistical theory and thermodynamics. Structural-scaling transitions. Thermalisation conditions.**

- **Generalization of Ginzburg-Landau approach, field equations, collective modes of defects, characteristic solid states (brittle, ductile, fine-grain states).**

- **Scaling and criticality of seismic events.**

- **Defect induced multiscale mechanisms of structural relaxation and non-linear responses of shocked materials.**

- **Discussion.**

Statistical mechanics of earthquakes

Conventional models can be classified as:

- **Dynamic models describing the momentum transfer (stress, velocity) along the existing faults)in the frame work of deterministic approach** (J. Rice, B. Kostrov);
- **Statistical models describing the probability of shear faults using the statistical data of seismic events** (J. Rundle, D. Turcotte et al.).

Physical model of earthquakes (J. Rundle et al., 1997)

- **System of localized shears** $s(x,t)$
- **Statistical equilibrium in the presence of stress**

$$\sigma_e\left[x,t,s(x',t'),p\right]=\sigma_f\left[x,t,s(x,t)\right],$$

where $\sigma_e(x,t)$ **is the elastic stress acting at the site on a fault due to the background traction p;** $\sigma_f(x,t)$ **is the frictional cohesive stress associated with the slip ;**

- **Probability distribution function for shears ensemble**

.

$$p(E_q)=\frac{1}{T_{sb}}\exp\left[-E_q/T_{sb}\right]$$

where T_{sb} **is the time average energy per blocks (effective temperature)**

$$T_{sb}=\frac{1}{2}\left\{\langle H(t)\rangle\right\}$$

Langevin equation for long-range modes

•**Energy functional** $H(s)$

$$H = \frac{1}{2}\sum_i \left\{ K_L (s_i)^2 + \frac{1}{2} K_C \sum_j [s_j - s_i]^2 \right\}$$

where K_L **and** K_C **are parameters**

Elastic stress as a function of "slip deficit"

$$\sigma_e[s] = \sigma_f[t] = \sigma_f[s + V\tau],$$

where V is group stress wave velocity.

Ito-Langevin equation

$$\frac{\partial s}{\partial \tau} = -\Gamma \frac{\delta U}{\delta s} + \eta(x, \tau),$$

where $\eta(x,\tau)$ **is the** δ **-correlated noise.**

$$\langle \eta(x,\tau)\eta(x',\tau') \rangle = 2\beta^{-1}\delta(\tau - \tau')\delta(x - x')$$

Solution of associated Fokker-Plank equation:

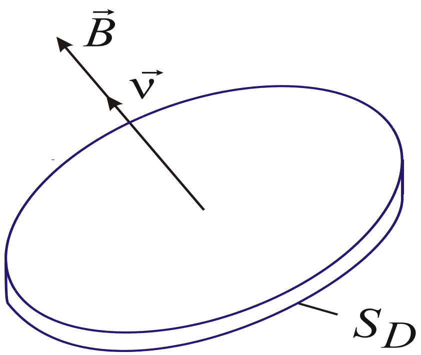
$$f[s] = Z^{-1} \exp\{-\beta U[s]\}$$

MICROSCOPIC VARIABLES FOR DEFECT ENSEMBLE

- Localization of Symmetry Groups of Distortion Tensor (**Gauge Field Theory**)

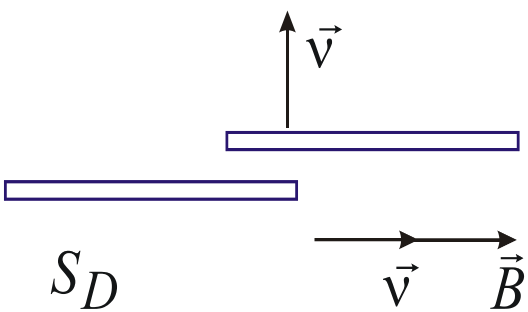
Microcrack

$$s_{ik} = s\nu_i \nu_k$$



Microshear

$$s_{ik} = \frac{1}{2} s \left(\nu_i l_k + l_i \nu_k \right)$$



Where $s = S_D B$ is the microcrack volume (shear intensity)

B is the Burgers vector.

STATISTICS OF MESODEFECTS

Generalization of the Boltzmann-Gibbs statistics for the “out-equilibrium” system (the Leontovich effective field method)

$$W = Z^{-1} \exp\left({}^{-E}\!/\!{}_{Q} \right),$$

Z is the normalization constant.

Statistical properties of the defect ensemble can be described after the determination of the defect energy *E* and the dispersion properties of the system given by the value of *Q*.

Microcrack (Microshear) Energy:

$$E = E_0 - H_{ik} s_{ik} + \alpha s_{ik}^2,$$

Effective Field:

$$H_{ik} = \sigma_{ik} + \lambda p_{ik} = \sigma_{ik} + \lambda n \langle s_{ik} \rangle .$$

CONSTITUTIVE EQUATION OF SOLID WITH MESODEFECTS

- Self-Consistency Equation for Defect Density Tensor

$$p_{i/k} = n \int s_{i/k} W(s, \vec{v}, \vec{l})\, ds_{ik}.$$

- Dimensionless Form

$$\tilde{p}_{ik} = \int \tilde{s}_{ik} Z^{-1} \exp\left((\tilde{\sigma}_{ik} + \frac{1}{\delta} \tilde{p}_{ik}) \tilde{s}_{ik} - \tilde{s}_{ik}^2 \right) d\tilde{s}_{ik}.$$

- General statistics

$$\tilde{p}_{ik} = \iint \tilde{s}_{ik} Z^{-1} \exp\left((\tilde{\sigma}_{ik} + \frac{1}{\delta} \tilde{p}_{ik}) \tilde{s}_{ik} - \tilde{s}_{ik}^2 \right) ds\, d\delta$$

- Dimensionless Material Parameter

$$\delta = \alpha / \lambda n.$$

$$\alpha \sim \frac{G}{V_0}, \quad \lambda \sim G, \quad n \sim R^{-3},$$

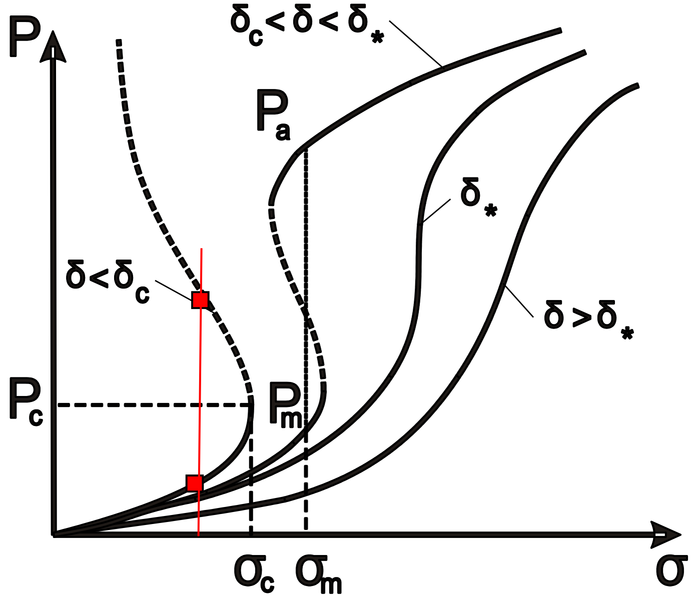
$$\delta \sim \left(R / r_0 \right)^3.$$

G is the elastic modulus, $V_0 \sim r_0^3$ is the defect nuclei volume,

R is the distance between defects.

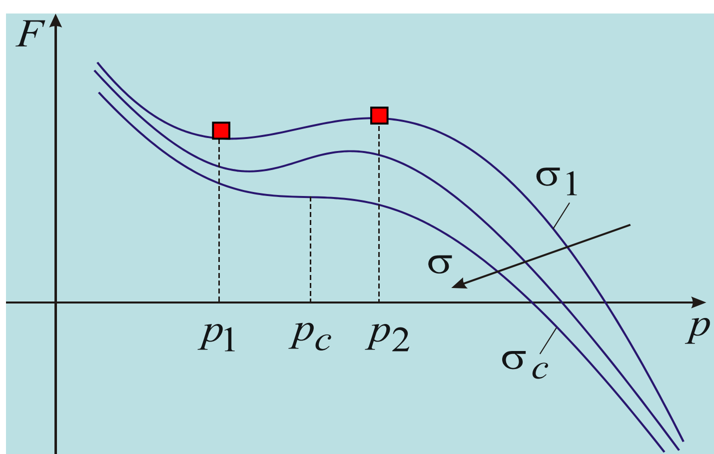
CHARACTERISTIC SOLID RESPONSES ON DEFECT GROWTH

- Solution of Self-Consistency Equation



- quasi-brittle $\delta<\delta_c=1$
- ductile $\delta_c<\delta<\delta_{¿}$
- fine grain state $\delta>\delta_{¿}\approx 1.3$

- Free Energy Dependence on Stress and Defect Density Tensor for $\delta<\delta_c$



PHENOMENOLOGY OF SOLIDS WITH MESODEFECTS

Non-Equilibrium Free Energy

Generalization of Ginzburg-Landau expansion (uni-axial case):

$$F = \frac{1}{2} A\left(\delta, \delta_{¿}\right) p^2 - \frac{1}{4} B p^4 - \frac{1}{6} C\left(\delta, \delta_c\right) p^6 - D\sigma\, p + \chi\left(\nabla_l p\right)^2 .$$

$$p = p_{zz},\ \sigma = \sigma_{zz},\ \varepsilon = \varepsilon_{zz}$$

- Gradient term $\chi\left(\nabla_l p\right)^2$ describes the non-local interaction in the defect ensemble (the long wave approximation);
- A, B, C, D are positive phenomenological material parameters;
- χ is the non-locality coefficient.

SELF-SIMILAR SOLUTIONS – COLLECTIVE MODES

- Solitary Wave:

$$\delta_c < \delta < \delta_{¿}$$

$$p(x,t) = \frac{1}{2} p_a \left[1 - \tanh\left(\xi\, l^{-1}\right)\right]$$

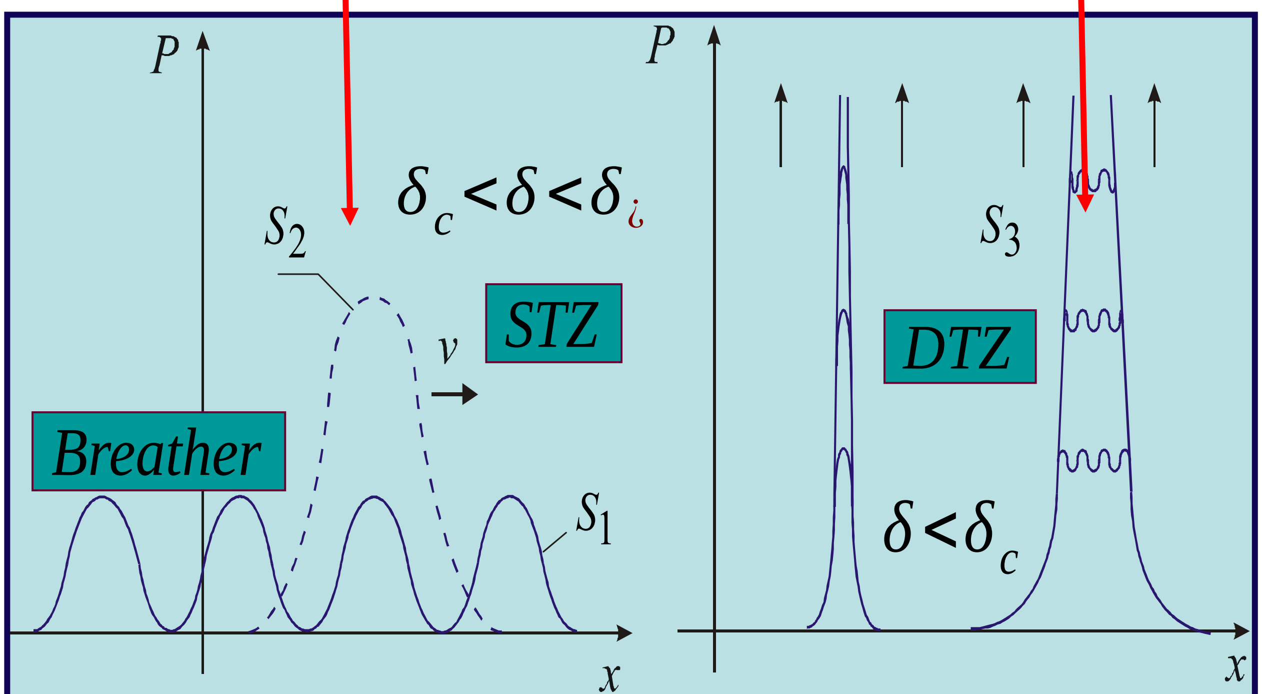
$$\xi = x - Vt, \quad V = \chi A\left(p_a - p_m\right) / \left(2 L_p^{-2}\right)$$

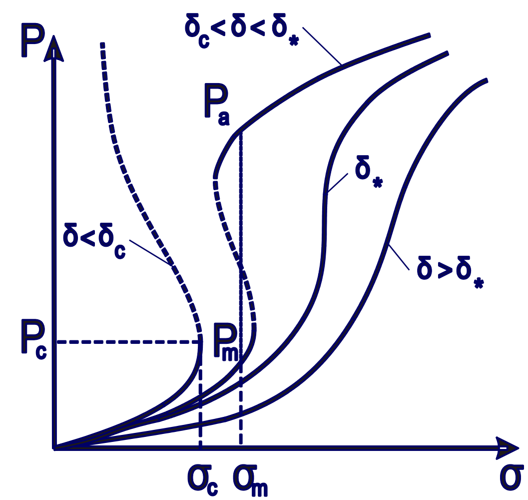
- ”Blow-up” Regimes of Damage Localization:

$$\delta < \delta_c$$

$$p(x,t) = \varphi(t) f(\zeta)$$

$$\zeta = {}^{x}\!/_{L_c}, \quad \varphi(t) = \Phi_0 \left(1 - \frac{t}{t_c}\right)^{-m}$$

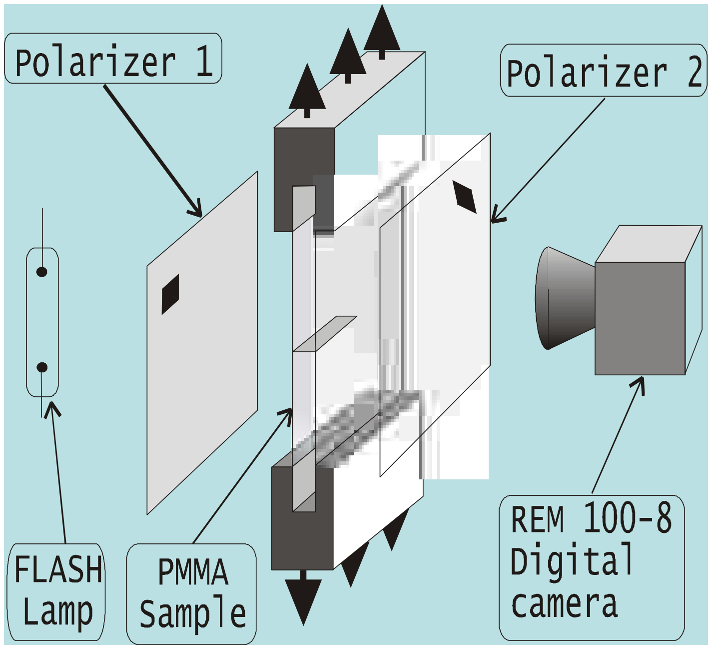


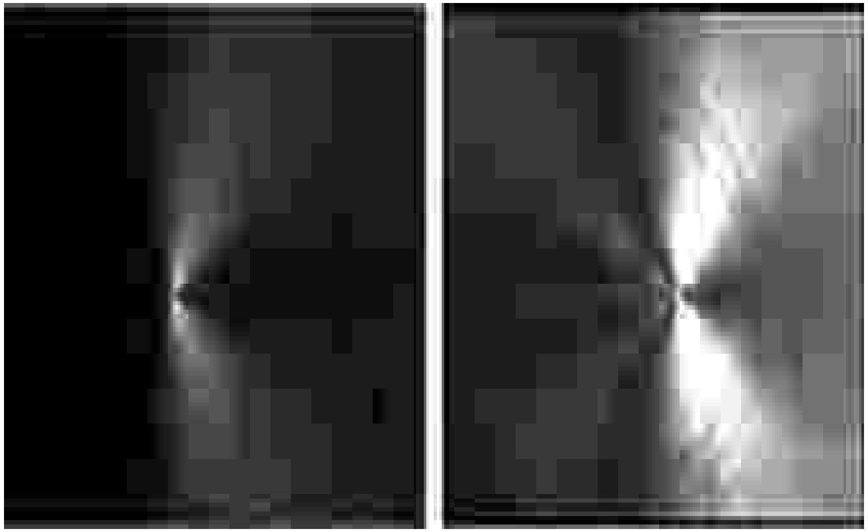


STATISTICS OF FAILURE AND FRAGMENTATION

- **EXPERIMENTAL STUDY OF NONLINEAR CRACK DYNAMICS :**

  High speed digital camera Remix REM 100-8, photo-elasticity method

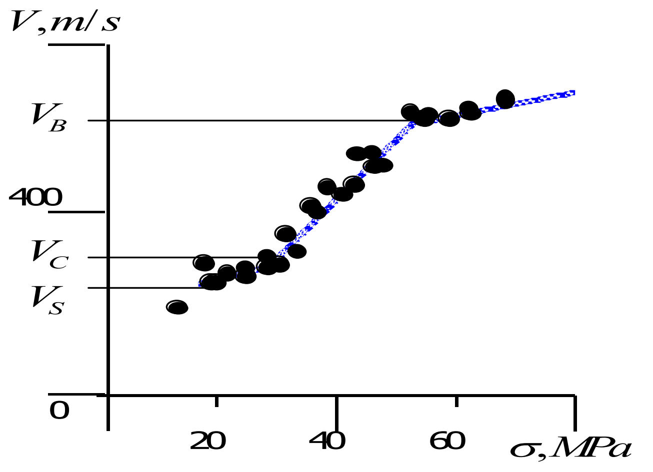
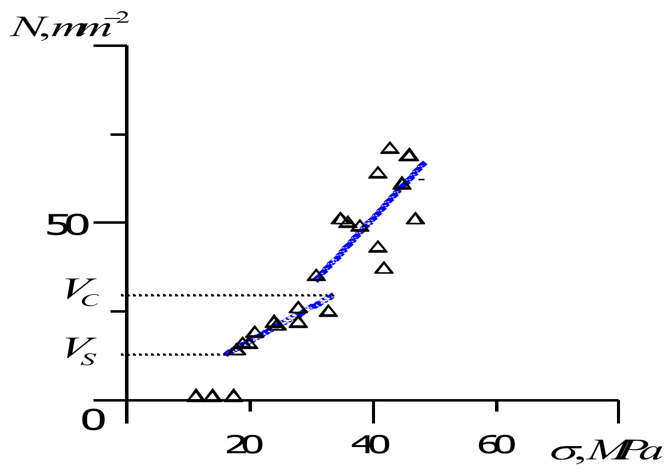




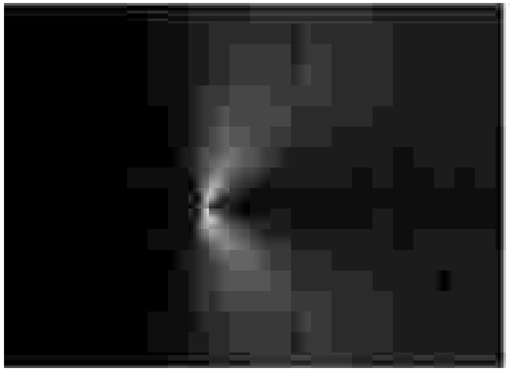
$V < V_C$ $V > V_C$

EXPERIMENTAL STUDY OF NONLINEAR CRACK DYNAMICS

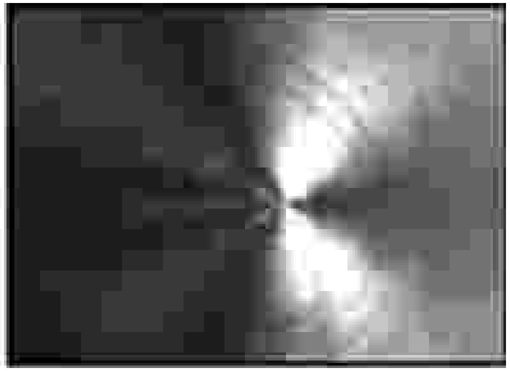
- Characteristic crack velocity

Crack velocity via initial stress	**Mirror zone concentration**
	

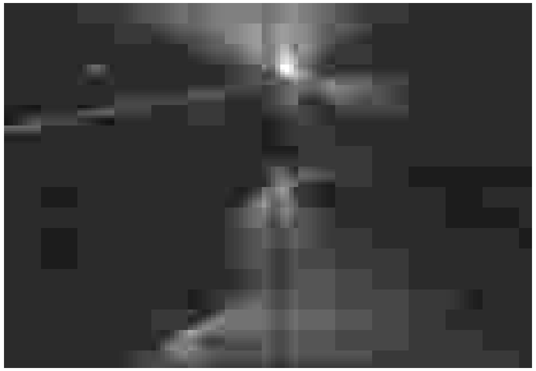
- Crack dynamics



$V<V_C$



$V>V_C$

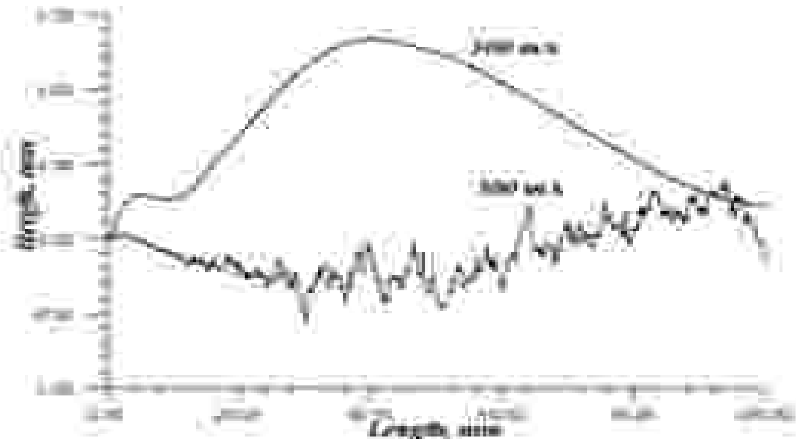


$V>V_B$

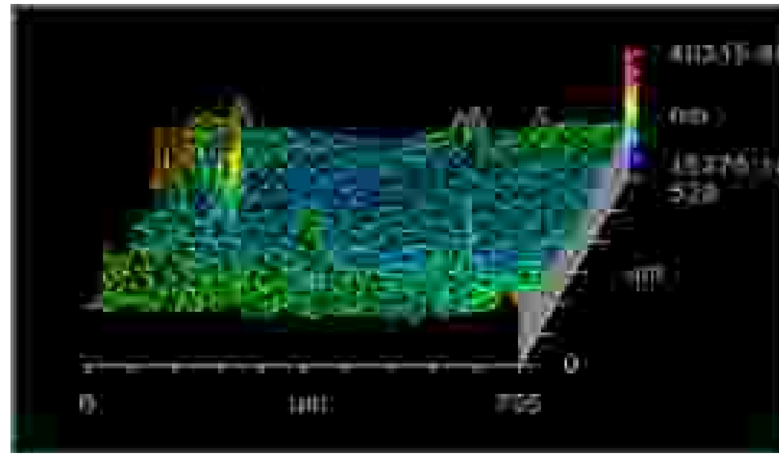
Scaling Analysis of Morphology of Failure Surface

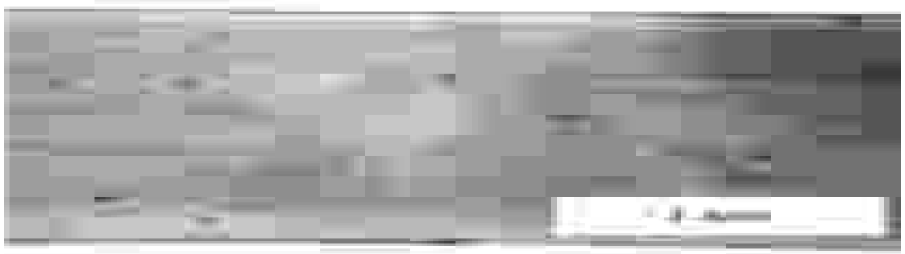
- Profilometry of PMMA Failure Surface

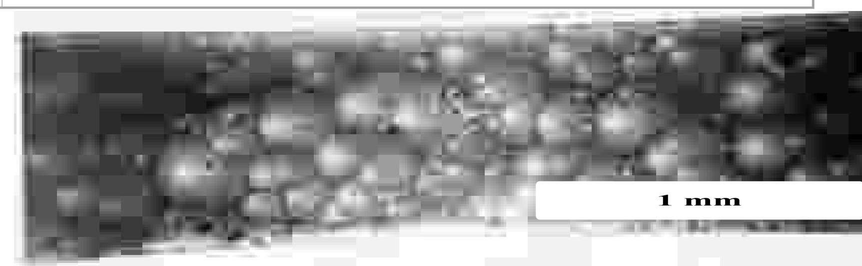
Profilometry of Surface



New View Digital Surface Profile





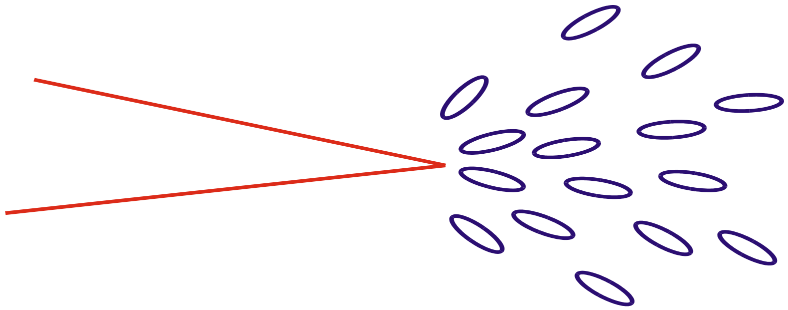


- Statistical Roughness Invariants - the Hurst Exponent

$$h(r) = \langle (z(r_0 + r) - z(r_0))^2 \rangle_r^{1/2} \sim r^{\zeta}$$

$$V = 500\ m/s,\ \zeta = 0.8$$

NONLINEAR CRACK DYNAMICS

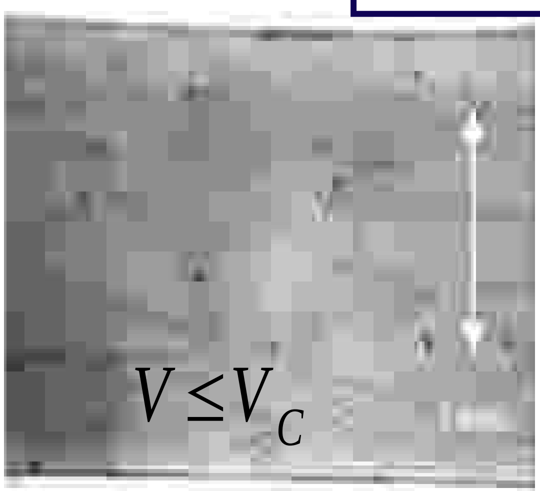


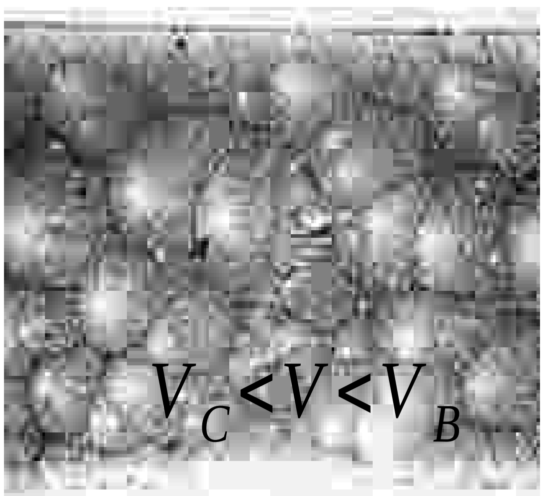
- **Self-similar solutions (attractor types)**

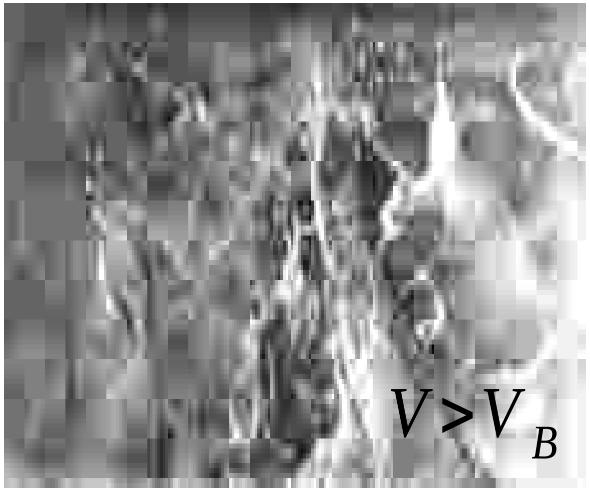
$$\sigma_{ik} \approx K_I r^{-\frac{1}{2}} f_{ij}(\theta) \qquad V \le V_C$$

$$p(x,t) = \varphi(t) f(\zeta), \quad \zeta = {}^{x}\!/\!{}_{L_c}, \; \varphi(t) = \Phi_0 \left(1 - \frac{t}{t_c}\right)^{-m} \qquad V > V_B$$

- **Critical velocity** $V_C = {}^{L_C}\!/\!{}_{t_c} \sim 300\, m/s,$ $\quad L_C \sim 0.3\, mm, \quad t_c \approx 1 \mu s$



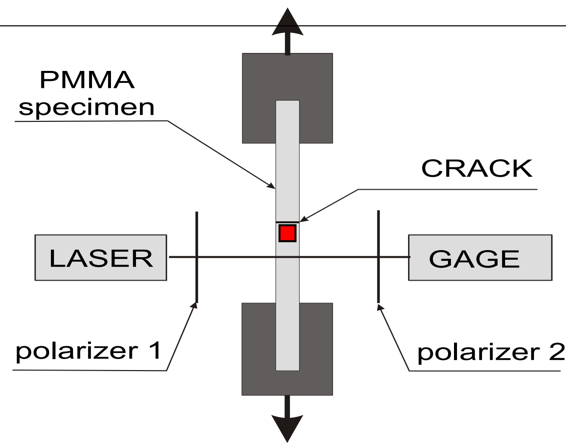




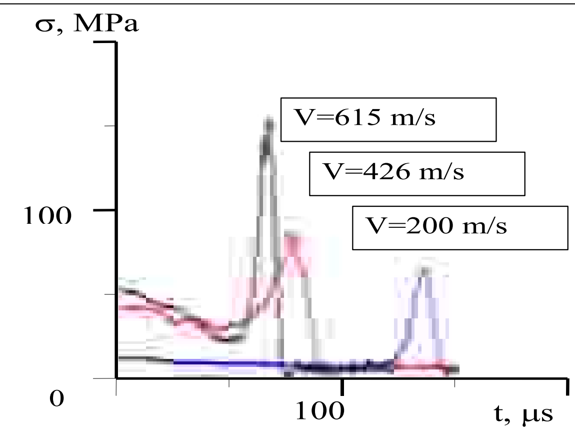
SCALING ANALYSIS OF ATTRACTOR TYPES FOR DYNAMIC VARIABLES

- **Failure under Dynamic Crack Propagation.**

● Experimental Set-Up

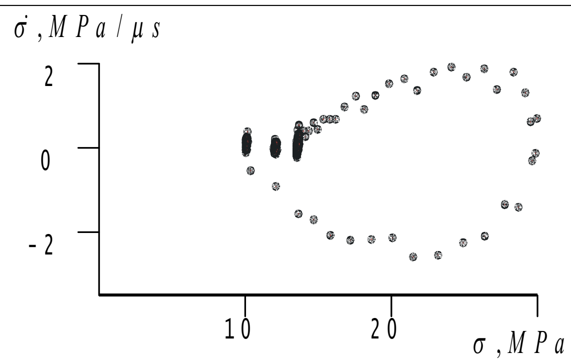


● Stress Dynamics

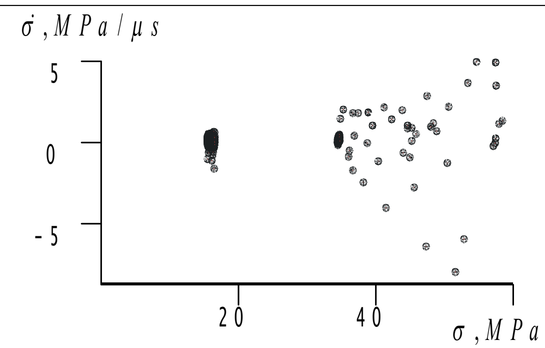


- **Attractor Types. Experimental Plots of the Poincare Cross Section**

● Steady State Crack Propagation

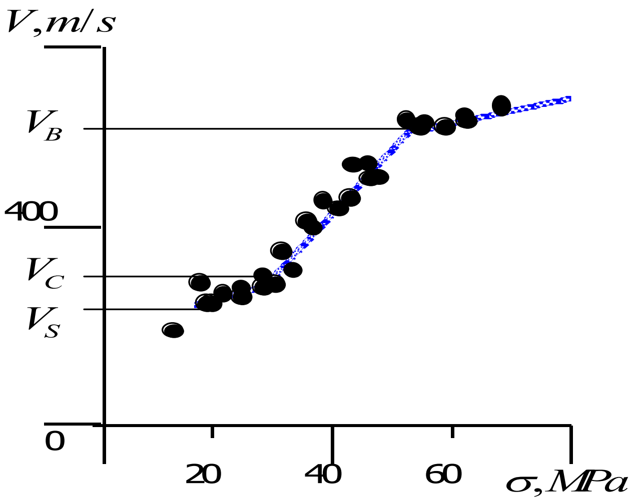


● Crack Branching



FRAGMENTATION STATISTICS

- Characteristic crack velocity



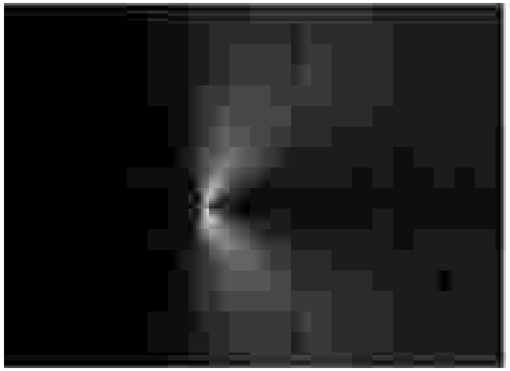
- Fragmentation scenario

$V_C < V$ - stress intensity controlled fragmentation scenario

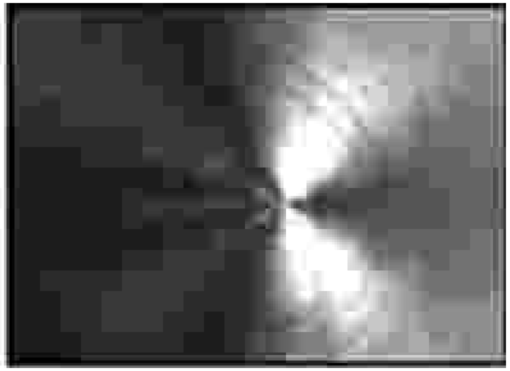
$V_C < V < V_B$ - intermediate (Weibull) statistics

$V > V_B$ -the Poisson statistics (limit case is mono-disperse statistics as failure wave precursor)

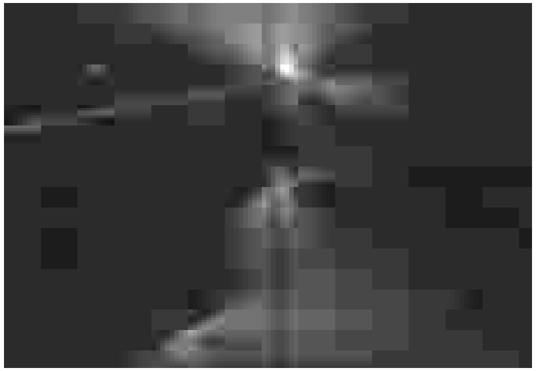
- Crack dynamics



$V < V_C$

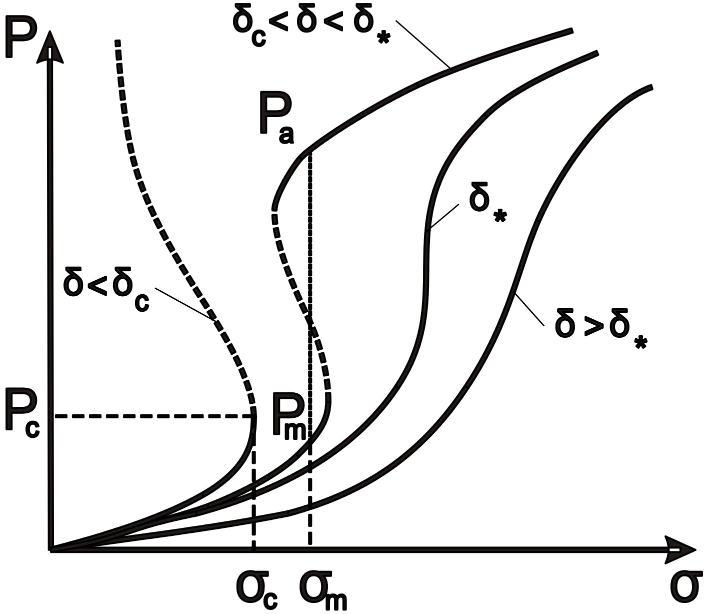


$V > V_C$



$V > V_B$

Earthquakes as blow-up regimes



Statement:
Seismic shocks is the consequence of generation of the blow-up self-similar collective modes in the slip ensemble (LS-режим).

$$\delta < \delta_c$$

$$\frac{dp}{dt} \approx S(p_c)p^{\omega} + \frac{\partial}{\partial x}\left(\chi_0(p_c)p^{\gamma}\frac{\partial p}{\partial x}\right)$$

There are three characteristic blow-up self-similar solutions:

1. **S-regime corresponds to the development of blow-up dissipative structure on the set of spatial (fundamental)length** L_f ;
2. **HS-regime corresponds to the development of expanding dissipative blow-up structure**;
3. **LS-режим corresponds to the generation of blow-up dissipative structures with a fundamental length, which depends on the non-linearity and the parameters of initial disturbances (the amplitude and spatial length).**

Kurdumov S.P. Evolution and self-organization laws of complex systems// International Journal of modern physics. 1988.-vol.1.-№4.

Earthquakes as blow-up regimes

PROBLEM 1: Kinetics of mesodefects ensemble in the vicinity of critical point p_C ($\delta < \delta_C$) for different initial p-distribution

$$\frac{dp}{dt} = S(p_c)p^{\sigma} + \frac{\partial}{\partial x}\left(\chi(p_c)p^{\beta}\frac{\partial p}{\partial x}\right)$$

$$\begin{cases} p(t,-L) = 0 \\ p(t,L) = 0 \end{cases} \quad p(0,x) = P_0(x)$$

$$\left.\frac{\partial p}{\partial x}\right|_{x=-L} = 0$$

$$\left.\frac{\partial p}{\partial x}\right|_{x=L} = 0$$

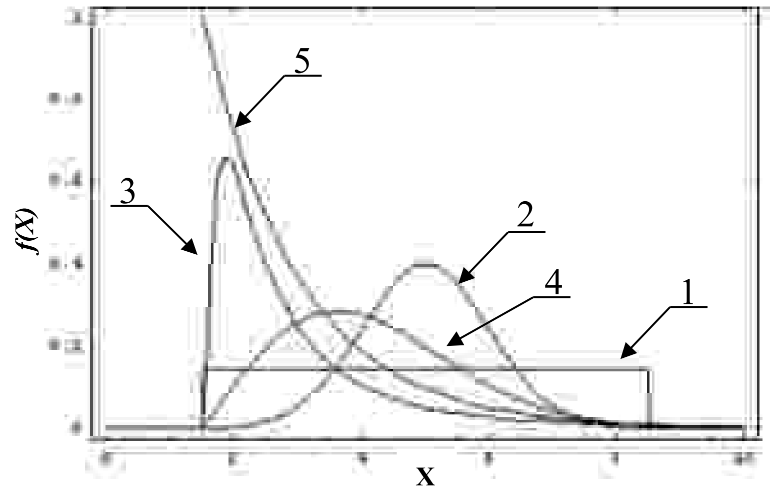
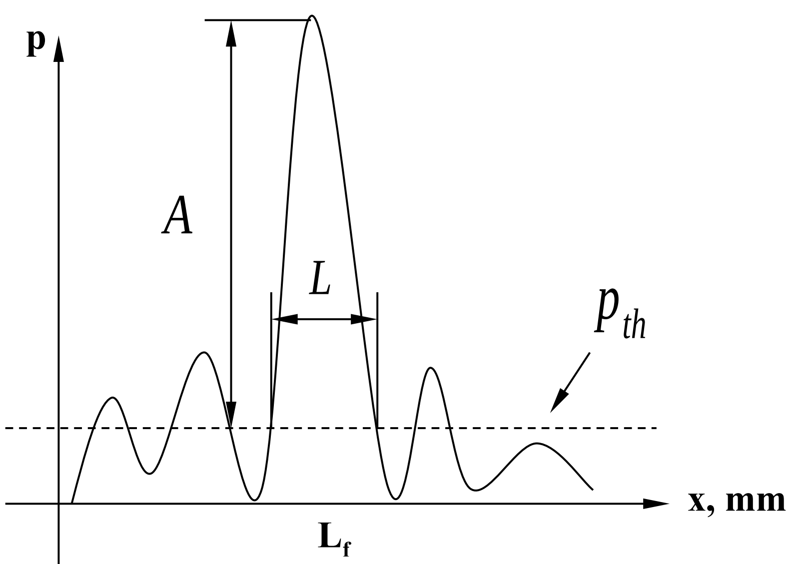


Fig.1. Initial p-distributions: (1 – uniform, 2 – Gaussian, 3 – Log-normal, 4 – Weibull, 5 – exponential)

Blow-up dissipative structure



A is the amplitude of blow-up structure

L is the length of blow-up structure

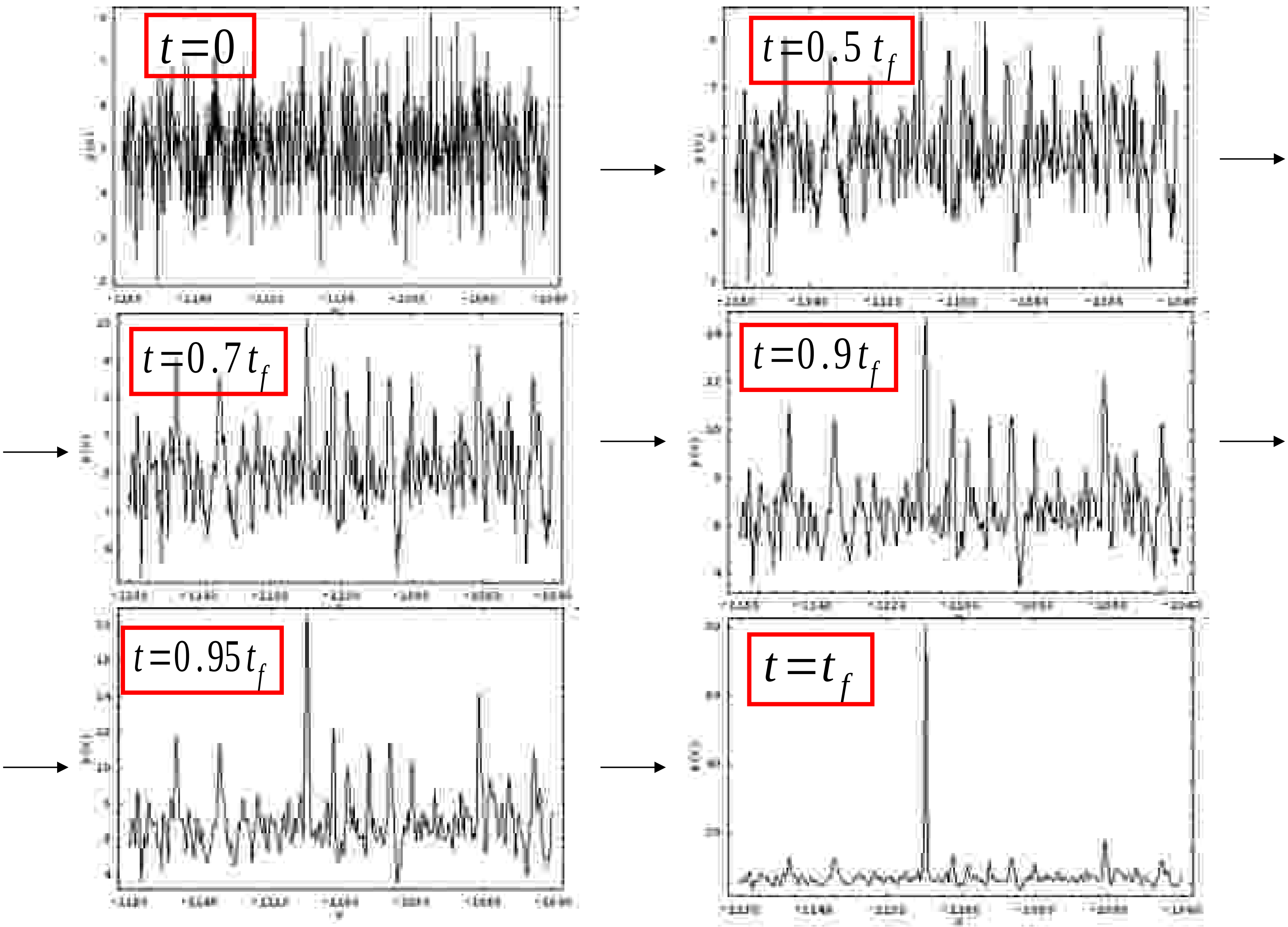
P_{th} is the threshold of p- registration

L_f is the fundamental length

Magnitude $M = \lg\left(\frac{A}{T}\right),\ T = \left(\frac{L}{v}\right)$

v is acoustic wave

Earthquakes as blow-up regimes



Scaling laws in seismicity

•**Gutenberg-Richter law** establishes self-similar features for earthquake frequency-magnitude data

$$N(¿m)=10^{a-bm}$$

Spatial scaling for earthquakes: $N=C\cdot A^{-D/2},\ D=2b$

where A is characteristic area of earthquake data, D is fractal dimension, D~2b.

•**Omori law** establishes the temporal decay of the rate of aftershocks following the mainshock

$$N=\frac{K}{(c-\tau)^p},$$

where K, c and p are parameters, p~1

•Modified Omori law describes the temporal decay of aftershocks, where c(m) is characteristic time refers to energy cascade

$$r(t,m)\equiv\frac{dN}{d\tau}=\frac{1}{\tau_{sc}[1+\tau/c(m)]^p}$$

• **Bath law** defines universal magnitude difference between the mainshock and aftershock with maximum magnitude

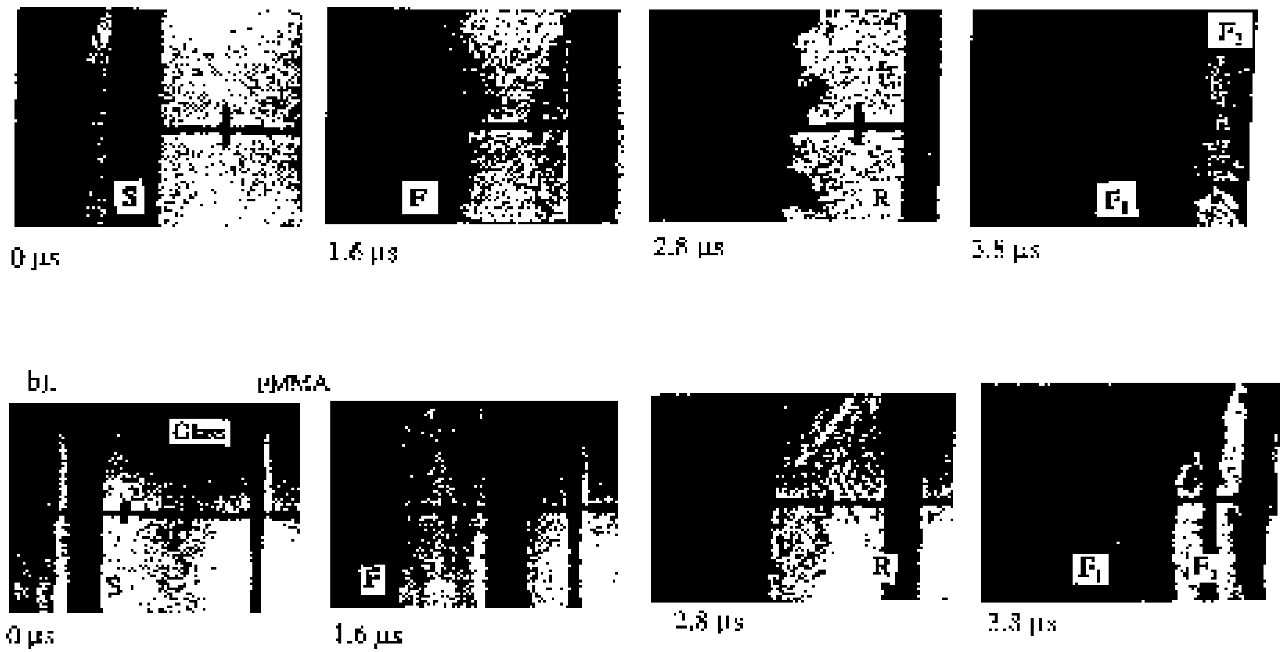
$$\Delta m = m_{ms} - m_{as}^{max} \approx 1.2$$

FAILURE WAVES

Rasorenov, S.V., Kanel, G.J., Fortov V.E. and Abasenov, M.M.(1991). High Pess. Res. 6, 225.

High Speed Framing of Shock Wave Propagation in Glass

(Bourne et al.. 1994)



Main Open Questions

How does a failure wave start?

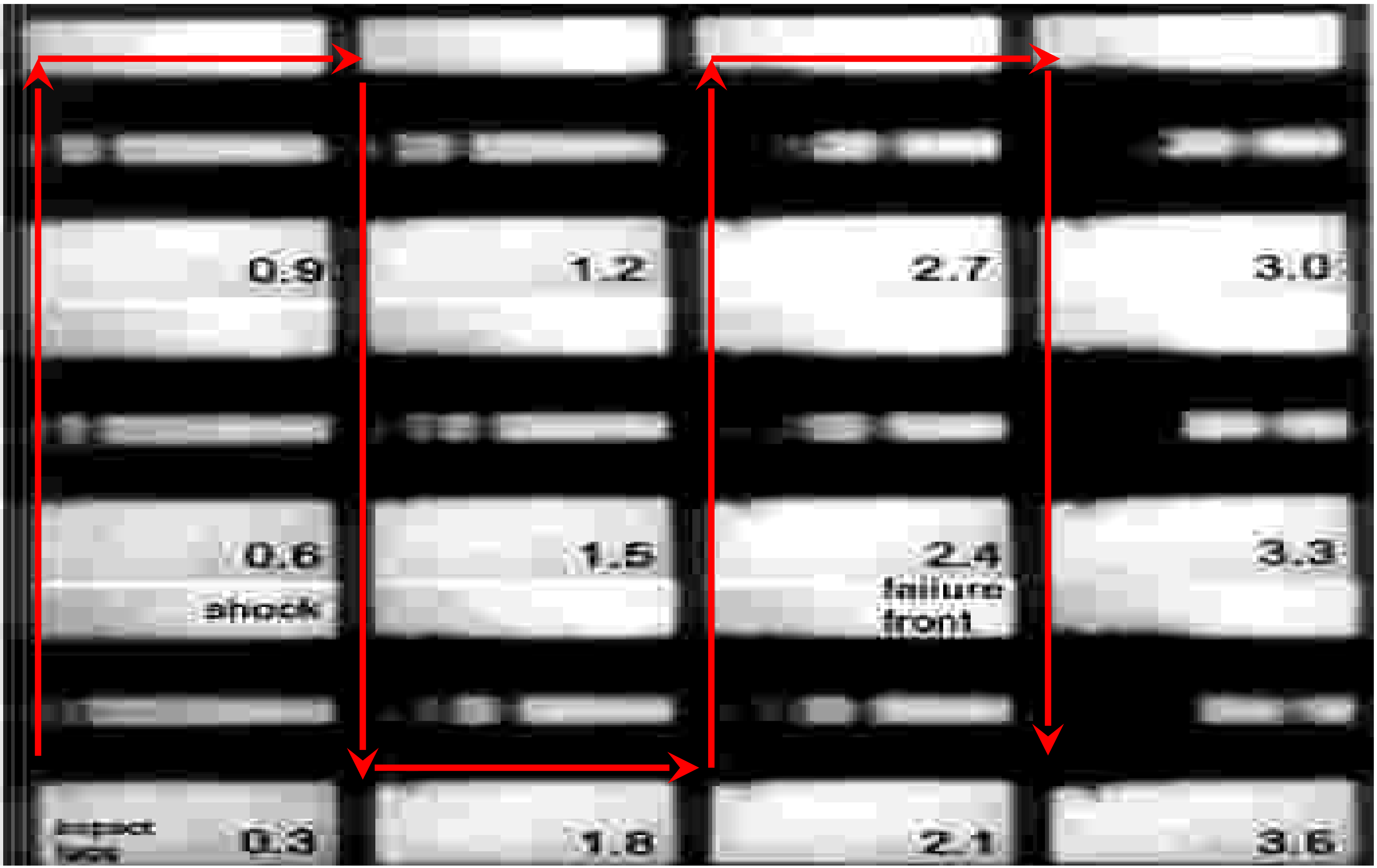
How does a failure wave propagate?

What is the material state behind a failure wave?

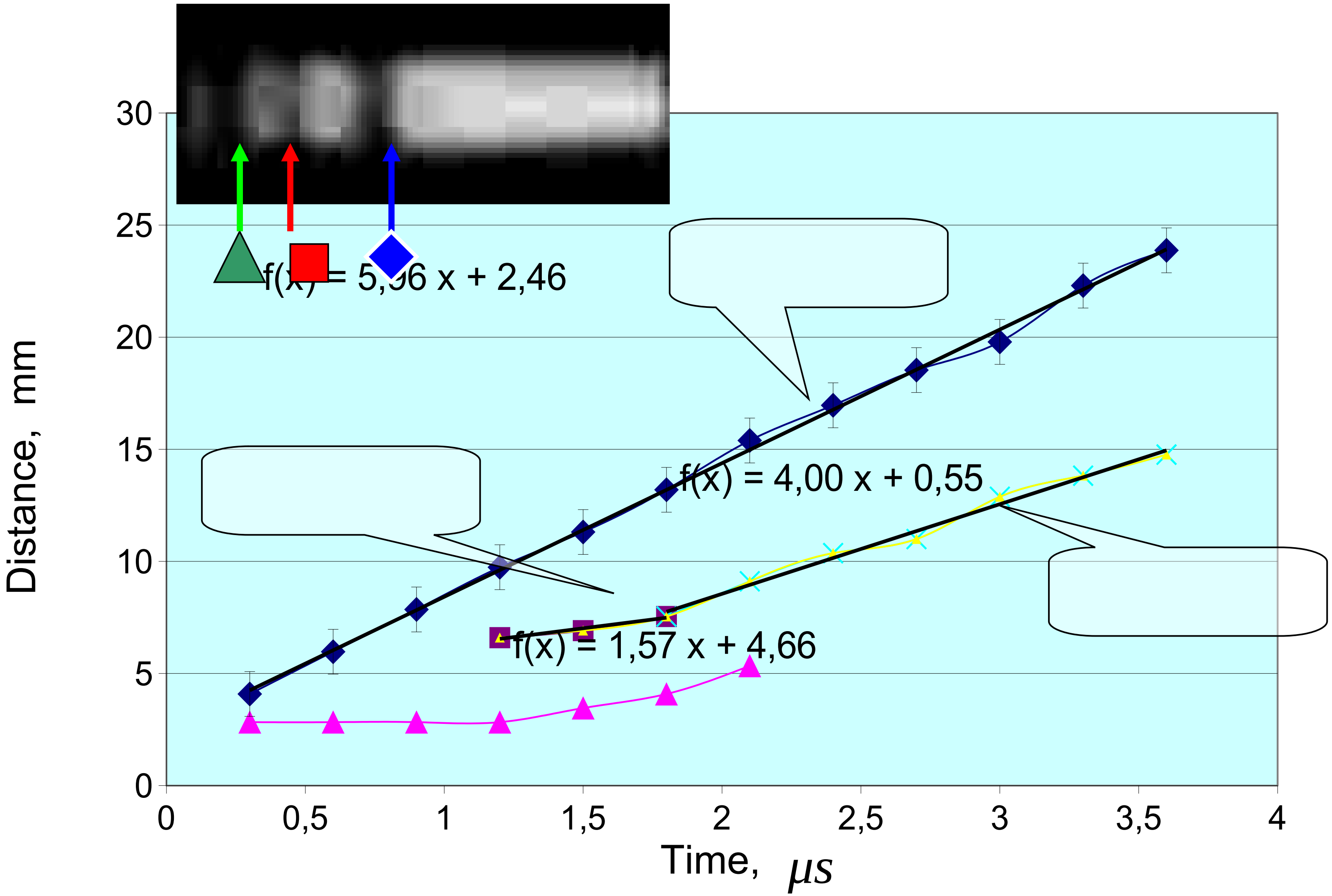
What are the kinetics of failure process and failure wave?

Symmetric Taylor Test for Fused-Quartz Rod

D.Radford, W.Proud, J.Field, O.Naimark et al., 2003



FRONT VELOCITIES



Salvador Dali's Self-organized Criticality