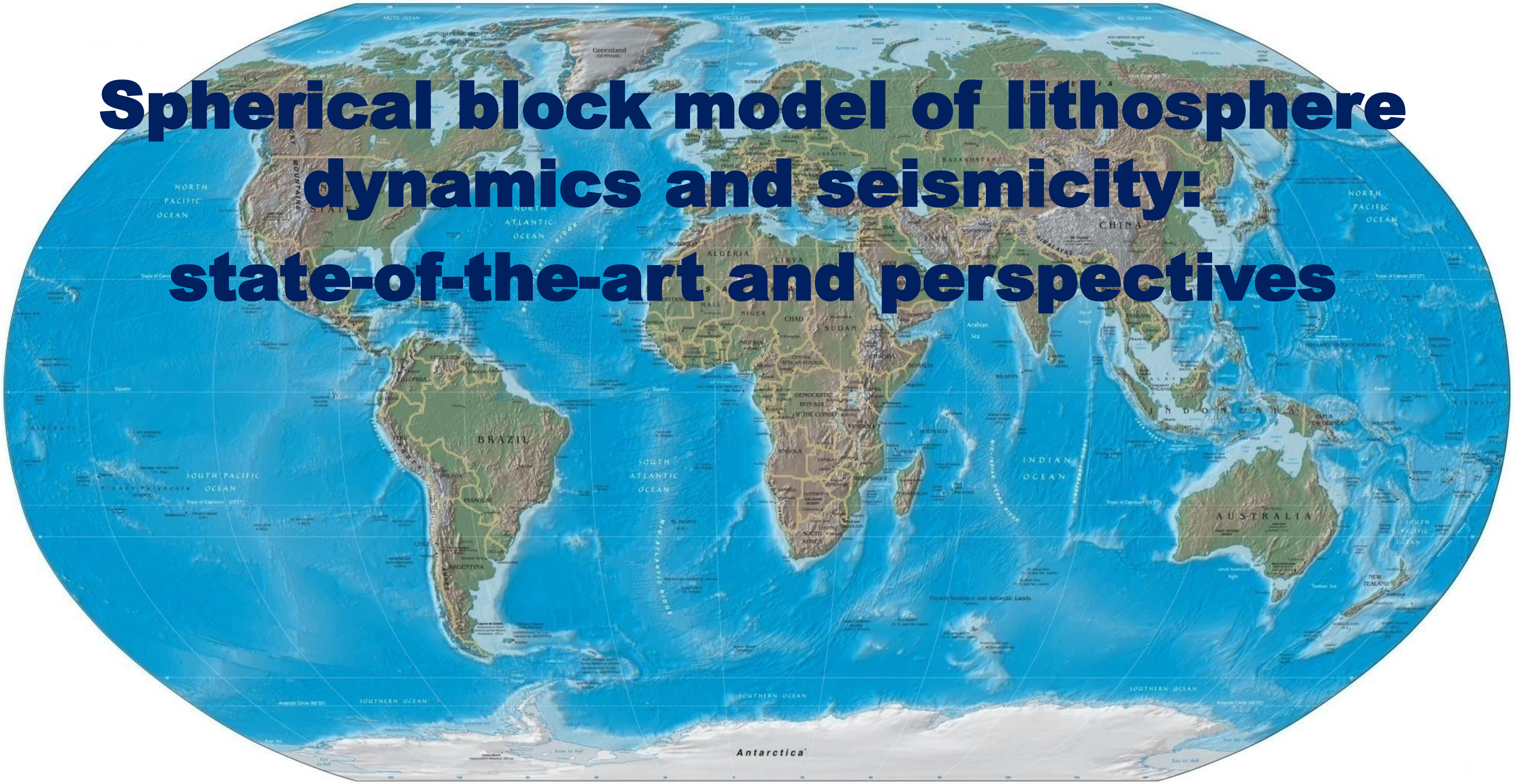
III All-Russian Scientific Conference with International Participation
MODERN METHODS OF SEISMIC HAZARD ASSESSMENT AND EARHTQUAKE PREDICTION – 2023
dedicated to the Memory of Professor A.A.Soloviev

October 25-26, 2023, International Institute of Earthquake Prediction Theory
and Mathematical Geophysics of RAS (Moscow, Russia)

Spherical block model of lithosphere dynamics and seismicity: state-of-the-art and perspectives



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Spherical block model of lithosphere dynamics and seismicity: state-of-the-art and perspectives

Outline:

1. The importance of seismicity simulation, approaches and goals.
2. The current modification of spherical block model: basic principles and possibilities.
3. Automatic procedure for calibrating model parameters. Appropriate criterion of simulation quality.
4. Numerical experiment: simulation of global seismicity. Optimal set of model parameters. Application of parallel technologies. Perspectives.

SPHERICAL BLOCK MODEL: MAIN REFERENCES

1. Digas B.V., Rozenberg V.L., Soloviev A.A., Sobolev P.O. Spherical model of block structure dynamics. Fifth Workshop on Non-Linear Dynamics and Earthquake Prediction, October 4-22, 1999, Trieste, Italy. 20 p.

2. Melnikova L.A. , Rozenberg V.L., Sobolev P.O., Soloviev A.A. Numerical simulation of dynamics of a system of tectonic plates: spherical block model. Comp. Seismology, M.: GEOS, 2000, Vol.31, pp.138-153.

3. Rozenberg V.L., Sobolev P.O., Soloviev A.A., Melnikova L.A. The spherical block model: dynamics of the global system of tectonic plates and seismicity. Pure and Appl. Geophys., 2005, Vol. 162, No. 1, p.145-164.

4. Melnikova L.A. , Rozenberg V.L., Soloviev A.A. Application of parallel technologies to simulation of lithosphere dynamics and seismicity. In: Supercomputer technologies in science, education, and industry. M.: MSU Press, 2010, pp.77-80.

5. Rozenberg V.L., Melnikova L.A., Soloviev A.A. Spherical block-and-fault model: basic principles, different modifications, and simulation of global seismicity. Advanced School on Understanding and Prediction of Earthquakes and other Extreme Events in Complex Systems, 26 September - 8 October, 2011, Trieste, Italy, SMR.2265-05, 42 p.

6. Melnikova L.A. , Rozenberg V.L. A stochastic modification of the spherical block-and-fault model of lithosphere dynamics and seismicity. Numerical Methods and Programming, 2015, Vol.16, pp.112-122.

7. Rozenberg V.L. Block model of lithosphere dynamics: new calibration method and numerical experiments. In: Sokolinsky L., Zymbler M. (eds) Parallel Computational Technologies. PCT 2020. Communications in Computer and Information Science, Springer, Vol.1263, pp.181-197.

FEATURES OF EARTHQUAKE SEQUENCES

(general for different tectonic structures and seismicity levels)

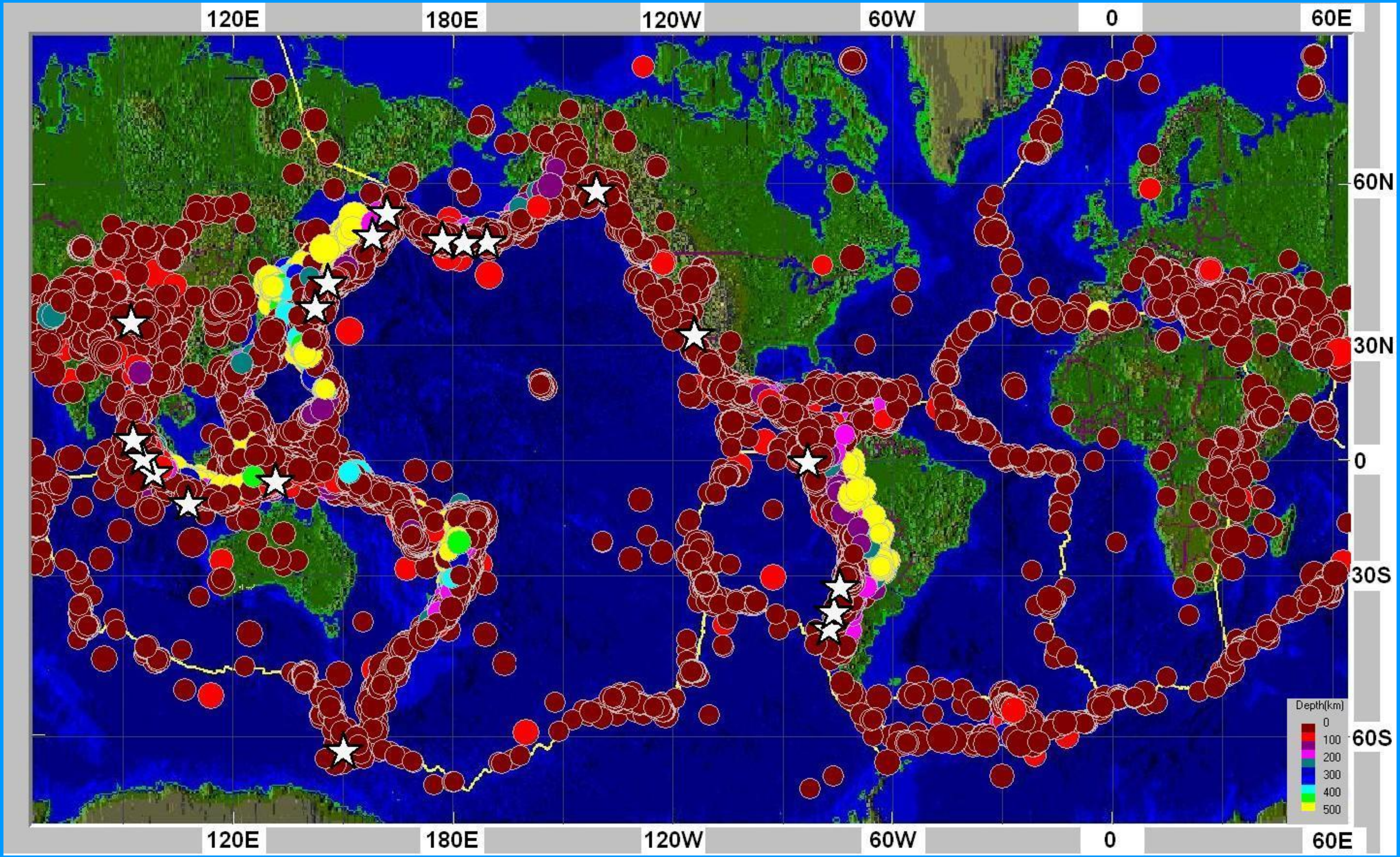
- Stationarity, quasi-periodicity, absence of noticeable trends.
- The Gutenberg—Richter frequency of occurrence law $\lg N(M) = a - bM$, where $N(M)$ is the distribution function of earthquakes above magnitude M, a and b are coefficients.
- The Omori law $n(t) = c / (1+t)^{\rho}$, where $n(t)$ is the number of aftershocks of a strong earthquake in t time units, $\rho \approx 1$.
- The migration of earthquakes along tectonic structures.
- Clustering (in space and time).
- The seismic cycle.
- Two temporal scales (slow: tectonic movements (cm/year); fast: abrupt stress release (km/sec)).

FREQUENCY OF OCCURENCE

descriptor	magnitude	average number per year	2021	2022
great	8 and more	1	3	0
major	7 – 7.9	15	16	11
strong	6 – 6.9	140	141	117
moderate	5 – 5.9	1457	2046	1603
light	4 – 4.9	≈ 13 000		
minor	3 - 3.9	≈ 130 000		
very minor	2 – 2.9	≈ 1 300 000		

NEIC (National Earthquake Information Center, USA) catalog data:
strong events from 1900,
weak events from 1990.

OBSERVED SEISMICITY
(catalog NEIC, events with M ≥ 6.0, 01.01.1900-31.12.2021)



Shallow events (depth < 50 km) are marked by **brown** circles, deeper events (<100 км), by **red**.
20 strongest events with M ≥ 8.4 are marked by white stars.

STRATEGIC GOALS OF SEISMICITY SIMULATION

- To reveal/to confirm precursors of extremal events.
- To analyze factors provoking earthquakes (e.g., tectonic driving forces or geometrical incompatibility of nodes), to evaluate competitive hypotheses.
- To study the migration of events, to determine possibly dangerous seismic regions.

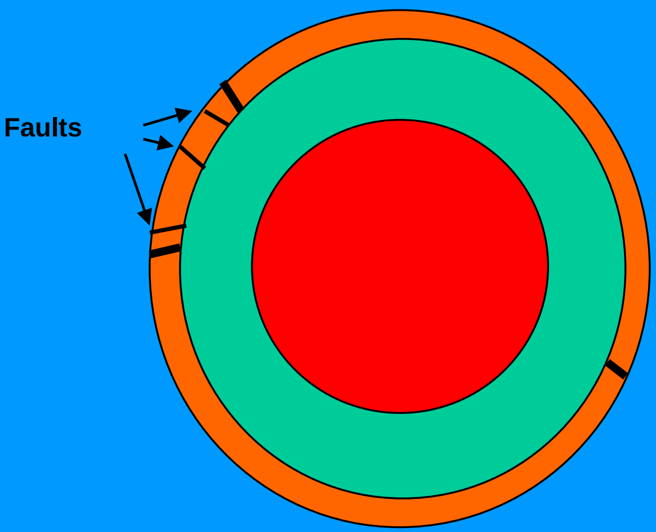
Adequate model as a tool incorporated into an expert system for seismic risk monitoring!!!

BASIC PRINCIPLES OF BLOCK MODELS

(V.I.Keilis-Borok, A.M.Gabrielov, A.A.Soloviev, 1986)

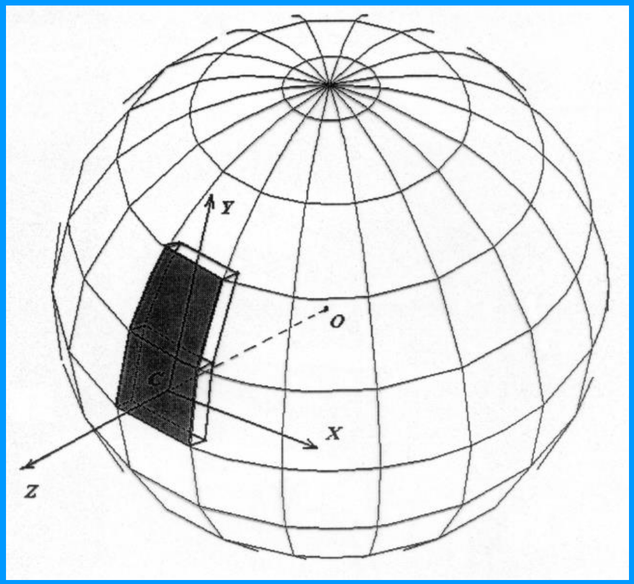
- a seismic region is represented as a **system of perfectly rigid blocks**, all deformations take place in the fault zones and at the block bottoms;
- the system of blocks moves as a consequence of the action of external forces applied to it (as a consequence of external motions of the underlying medium and boundary blocks);
- the system is supposed to be in the **quasistatic equilibrium state** at every time instant;
- all displacements are small comparing with block sizes, the geometry of the structure does not change in the process of simulation;
- no influence of gravity forces;
- three types of interaction between blocks are considered: **visco-elastic, stress-drop, creep**.

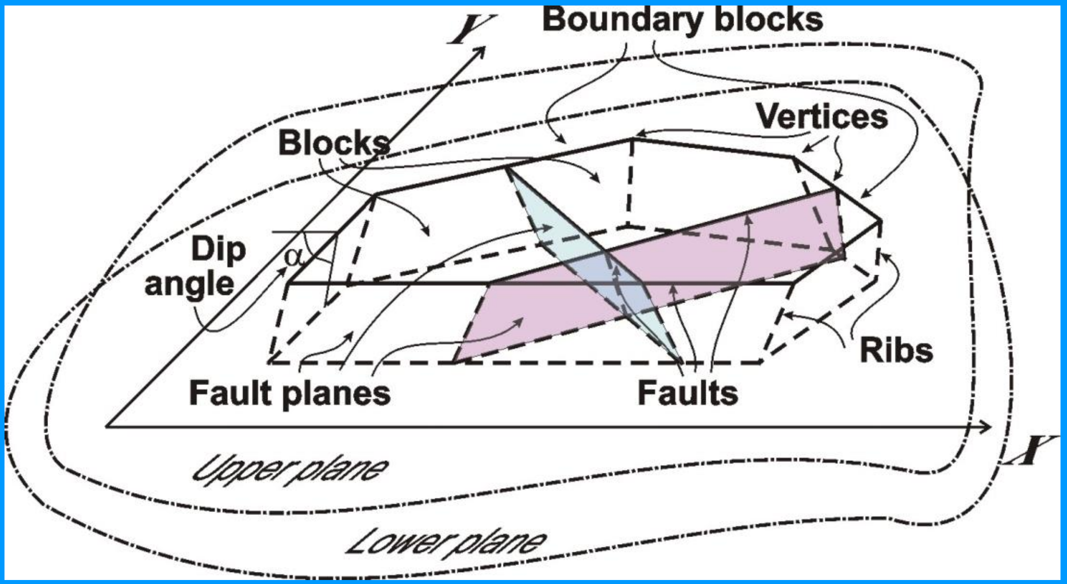
BLOCK STRUCTURE



A **block structure** is a limited and simply connected part of a spherical layer of a depth *H* bounded by two concentric spheres. The outer sphere represents the Earth's surface and the inner one represents the boundary between the lithosphere and the mantle.

Partition of the structure into **blocks** is defined by **faults** intersecting the layer. Each fault is a part of a conic surface with a **dip angle**α.





INTERACTION BETWEEN BLOCKS

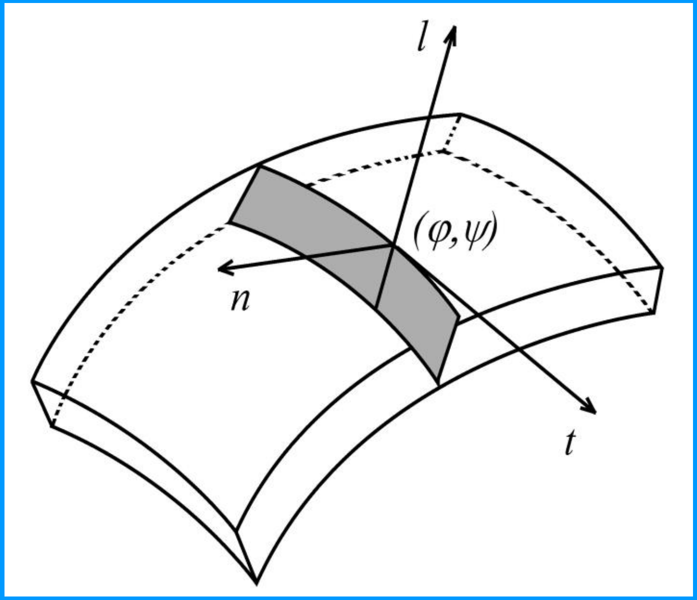
The elastic force (f_t, f_l, f_n) per unit area of fault applied to point (φ, ψ) at time τ:

$f_t(\tau) = K_t(\Delta_t(\tau) - \delta_t(\tau))$, $f_l(\tau) = K_l(\Delta_l(\tau) - \delta_l(\tau))$, $f_n(\tau) = K_n(\Delta_n(\tau) - \delta_n(\tau))$.

The evolution of inelastic displacements δ_t, δ_l, δ_n:

$$d\delta_t(\tau) = W_t K_t(\Delta_t(\tau) - \delta_t(\tau))d\tau + \lambda_t \delta_t(\tau)\, d\xi_t(\tau),$$
$$d\delta_l(\tau) = W_l K_l(\Delta_l(\tau) - \delta_l(\tau))d\tau + \lambda_l \delta_l(\tau)\, d\xi_l(\tau),$$
$$d\delta_n(\tau) = W_n K_n(\Delta_n(\tau) - \delta_n(\tau))d\tau + \lambda_n \delta_n(\tau)\, d\xi_n(\tau).$$

Here, Δ_t, Δ_l, Δ_n are components of relative displacements in system (t, l, n), ξ_t, ξ_l, ξ_n are standard independent scalar Wiener processes, λ_t, λ_l, λ_n are amplitudes of random actions, coefficients K_t, K_l, K_n, W_t, W_l, W_n determine visco-elastic properties of fault and may be different for different faults.



EQUILIBRIUM EQUATIONS

Displacements of blocks are found from the condition:

for each block **the total force and the total moment of forces acting on it are equal to zero**. This is a condition of the **quasi-static equilibrium** of the system (and the condition of energy minimum).

The dependence of forces and their moments on displacements of blocks is linear.

Therefore, the equilibrium is described by the system of linear equations for components of translation vectors of blocks and angles of their rotation:

$$A\boldsymbol{w} = \boldsymbol{b}.$$

The components of unknown vector $\boldsymbol{w} = (w_1, w_2, \ldots, w_{6n})$ are components of translation vectors of blocks and angles of their rotation (n is the number of blocks).

EARTHQUAKE AND CREEP

The levels $B > H_f > H_s$ are specified for each fault :

$$B = B(\tau_i) = B_0(\tau_i) + \sigma X(\tau_i), \quad H_f = H_f(\tau_i) = a\,B(\tau_i), \quad H_s = H_s(\tau_i) = b\,B(\tau_i).$$

At every time τ_i, the value of $\boldsymbol{\kappa}$ (model stress) is calculated for all cells:

$$\kappa = \frac{\sqrt{f_t^2 + f_l^2}}{P - f_n}.$$

Here, P is the parameter that is equal for all faults and can be interpreted as the difference between lithostatic and hydrostatic pressure. If at time τ_i $\boldsymbol{\kappa} \geq B$ for some cell, then, in accordance with the dry friction model, a **failure** (an **"earthquake"**) occurs. By a failure is meant a slippage by which the inelastic displacements in the cell change abruptly to reduce the value of $\boldsymbol{\kappa}$ to the level H_f. Then, the cell is in the **creep state** until $\boldsymbol{\kappa}$ reaches the level H_s.

EARTHQUAKE PARAMETERS

(i) **time** τ_i;

(ii) the **epicentral coordinates** and the **depth** are the weighted sums (weights are proportional to the areas of the cells) of the coordinates and depths of the cells involved in the earthquake

(iii) the **magnitude** is calculated by (Wells and Coppersmith, 1994):

$$M = D \lg S + E,$$

where S is the total area of quaked cells (in km^2), D and E are empirical constants depending on the quake mechanism.

A **synthetic earthquake catalog** is a basic result of numerical simulation, every model event is characterized by origin time, epicentral coordinates, depth, and magnitude

CALIBRATION OF THE MODEL

- Spatial distribution of epicenters of strong events, the Hausdorff metric:

$$d_{\mathrm{H}}(E_r, E_m) = \max\{ \sup_{e_r \in E_r} \inf_{e_m \in E_m} d(e_r, e_m), \sup_{e_m \in E_m} \inf_{e_r \in E_r} d(e_r, e_m) \},$$

E_r, E_m are the sets of real and model epicenters,

$d(e_r, e_m)$ is the distance between points on the Earth's surface.

- Distribution of earthquakes in depth:

$$d_{\mathrm{D}}(D_r, D_m) = \sum_{i=1}^{n} |D_r^i - D_m^i|,$$

D_r, D_m are real and model data,

index *i* corresponds to a share of events from *i* th interval in depth.

- Parameters of the Gutenberg-Richter law:

$$d_{\mathrm{G}}(G_r, G_m) = \alpha_1 |S_r - S_m| + \alpha_2 |A_r - A_m|,$$

G_r, G_m are real and model parameters,

S is an estimate of the slope of regression $\lg N = c - SM$, *A* is an averaged approximation error.

CALIBRATION OF THE MODEL

An aggregated criterion of simulation quality:

$$Er_j = \beta_1 d_{\mathrm{H}_j} / \max_i d_{\mathrm{H}_i} + \beta_2 d_{\mathrm{D}_j} / \max_i d_{\mathrm{D}_i} + \beta_3 d_{\mathrm{G}_j} / \max_i d_{\mathrm{G}_i} \to \min_j,$$

Er_j is aggregated error of variant j, $j=1,\dots,K$, K is the number of variants under examination, $\beta_1, \beta_2, \beta_3 > 0$, $\beta_1 + \beta_2 + \beta_3 = 1$.

NB: *Er* = 0 for real data and can be equal to 1 for the worst variant.

Conception of the numerical experiment:

- The choice of key model parameters, intervals and steps of their change, construction of the set of *K* variants to find an optimal one.
- Automatization and parallelization of the launch process and of the analysis of simulation results. Study of the optimal variant.

NECESSITY OF PARALLELIZATION

Computational experiments show that the spherical block model of lithosphere dynamics and seismicity during performing on sequential computers requires considerable expenditures of memory and processor time (CPU 3.6 GHz: more than 24 h for a variant of 100 model time units).

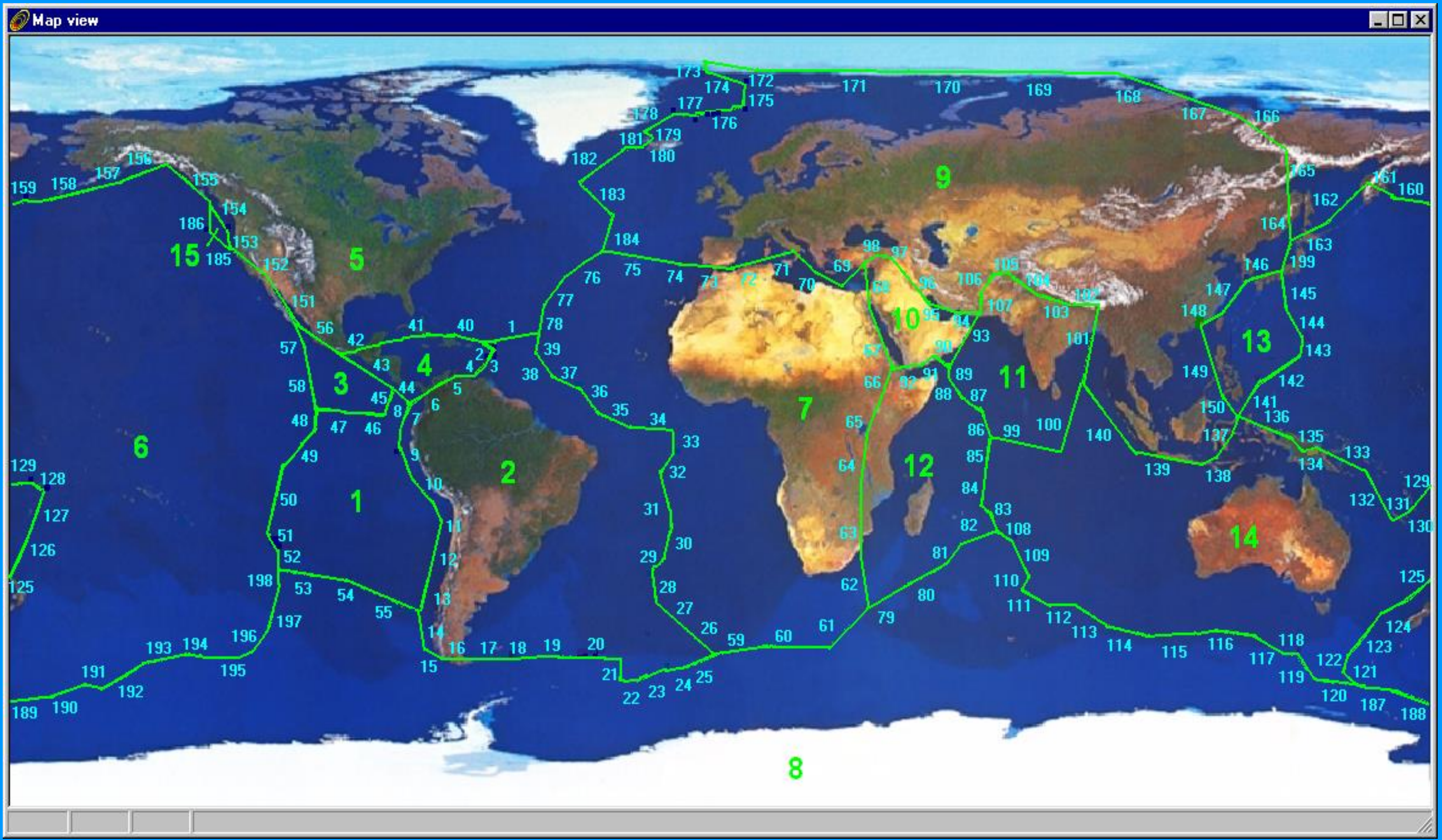
BUT

the approach applied to modeling admits effective parallelization of calculations on a multiprocessor cluster, and it makes possible the use of real geophysical and seismic data in the process of simulation of dynamics of complicated block structures, including the global system of tectonic plates.

Hybrid supercomputer «URAN» (1940 CPU Intel Xeon and 314 GPU NVIDIA Tesla, peak performance is about 215 TFlops):

50 processors, **acceleration coefficient $S_r = T_1 / T_r = 45$**

THE BLOCK STRUCTURE APPROXIMATING THE GLOBAL SYSTEM OF TECTONIC PLATES



THE BLOCK STRUCTURE APPROXIMATING THE GLOBAL SYSTEM OF TECTONIC PLATES

Blocks / Plates:

1 - Nazca (depth 50 km), 2 - South America (10 km),
3 - Cocos (50 km), 4 - Caribbean (10 km),
5 - North America (10 km), 6 - Pacific (100 km),
7 - Africa (10 km), 8 - Antarctica (10 km),
9 - Eurasia (30 km), 10 - Arabia (10 km),
11 - India (50 km), 12 - Somalia (10 km),
13 - Philippines (50 km), 14 - Australia (50 km),
15 - Juan de Fuca (50 km).

Totally: 15 blocks, 186 vertices, 199 faults (segments).

Total number of time steps:		up to 1 000 000
Total number of cells:		
	on segments	about 3 500 000
	on block bottoms	about 250 000

Reference:

V. Rozenberg, P. Sobolev, A. Soloviev, and L. Melnikova. The Spherical Block Model: Dynamics of the Global System of Tectonic Plates and Seismicity, Pure appl. geophys., 2005, V.162, pp.145-164.

EXPERIMENT PARAMETERS

Grid with respect to model parameters of two types.

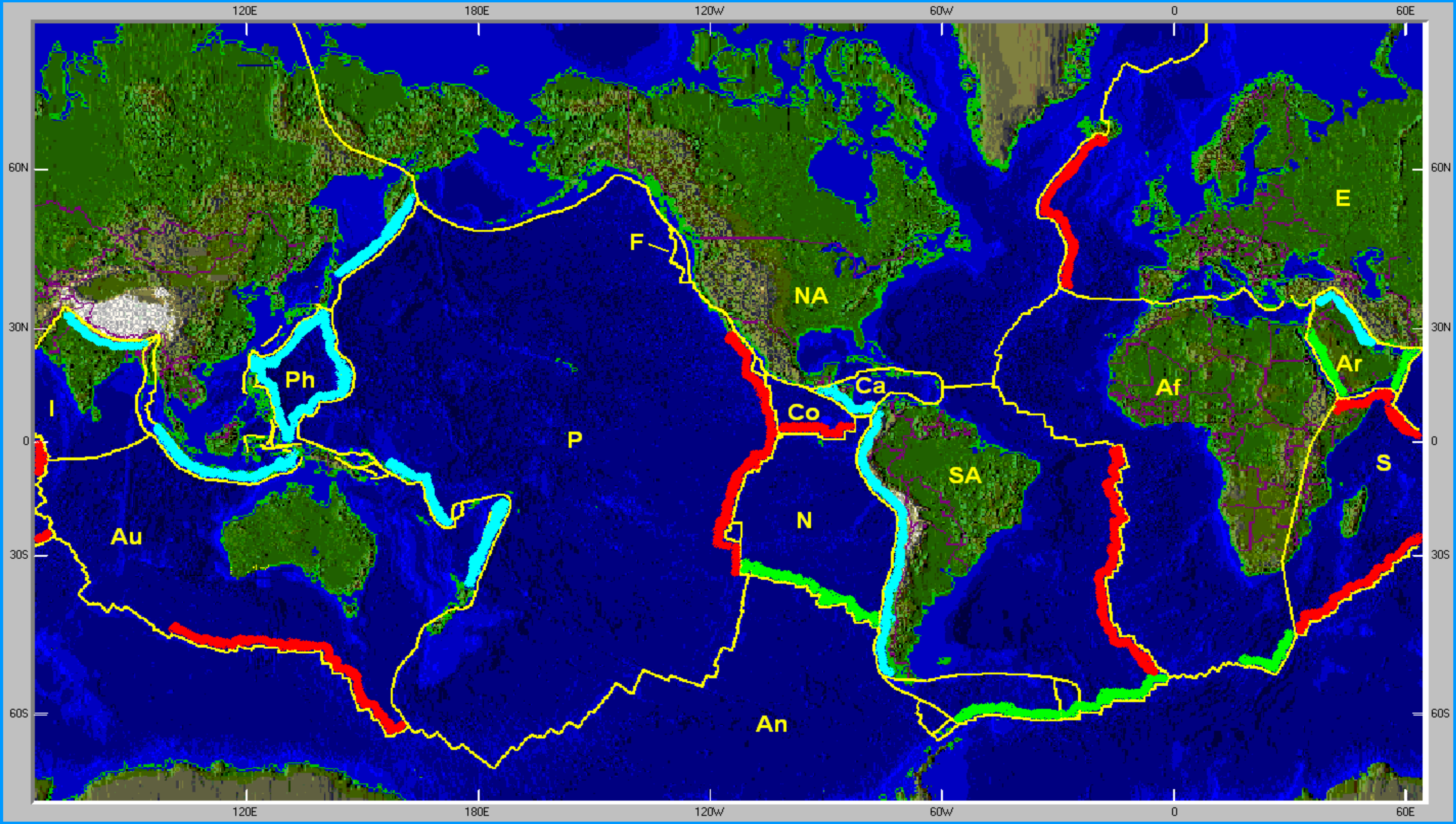
- Characteristics of visco-elastic interaction along faults:
  $K_t, K_l, K_n \subset [1, 10]$, $\Delta_K=1$, $W_t, W_l, W_n \subset [0.01, 0.05]$, $\Delta_w=0.01$,
  in creep state $W_t, W_l, W_n \subset [5, 10]$, $\Delta_w=5$.
- Characteristics of random factors:
  $\lambda_t, \lambda_l, \lambda_n \subset [0, 0.1]$, $\Delta_\lambda=0.05$, $B_0 \subset [0.1, 0.2]$, $\Delta_B=0.1$, $\sigma \subset [0, 0.05]$, $\Delta_\sigma=0.05$.

Total: 10 x 5 x 2 x 3 x 2 x 2 = 1200 variants.
$d_{Hmax}(E_r, E_m) = 5360$ km, $d_{Dmax}(D_r, D_m) = 0.66$, $d_{Gmax}(G_r, G_m) = 1.1$.

Optimal set of parameters minimizing the criterion:

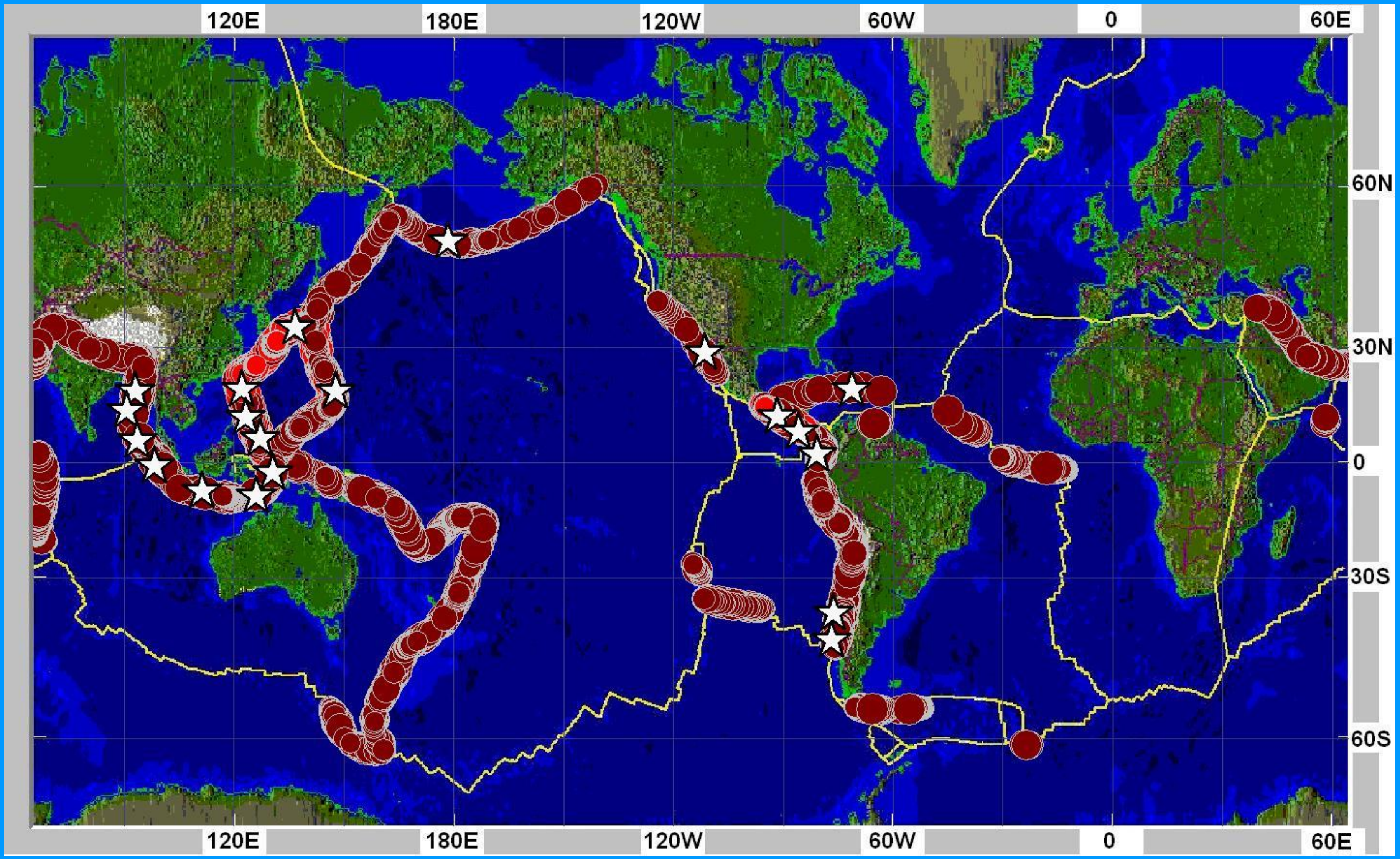
$K_t = K_l = K_n = 8$,
$W_t = W_l = W_n = 0.02$, in creep state $W_t = W_l = W_n = 10$,
$\lambda_t = \lambda_l = \lambda_n = 0.1$,
$B_0 = 0.1$,
$\sigma = 0$.
$d_H(E_r, E_m) = 3450$ km, $d_D(D_r, D_m) = 0.36$, $d_G(G_r, G_m) = 0.125$.
$Er = 0.43$.

THE RESULTS OF SIMULATION OF THE PLATE BOUNDARIES CHARACTER



The plate boundaries: divergent (extension, red), convergent (subduction, rlight blue), transform (shift, green).

THE SYNTHETIC SEISMICITY
(optimal variant, 100 units of model time)



Shallow events (depth < 50 km) are marked by **brown** circles, deeper events (<100 км), by **red**.
20 strongest events with M ≥ 8.6 are marked by white stars.

THE SYNTHETIC SEISMICITY
(optimal variant, 100 units of model time)

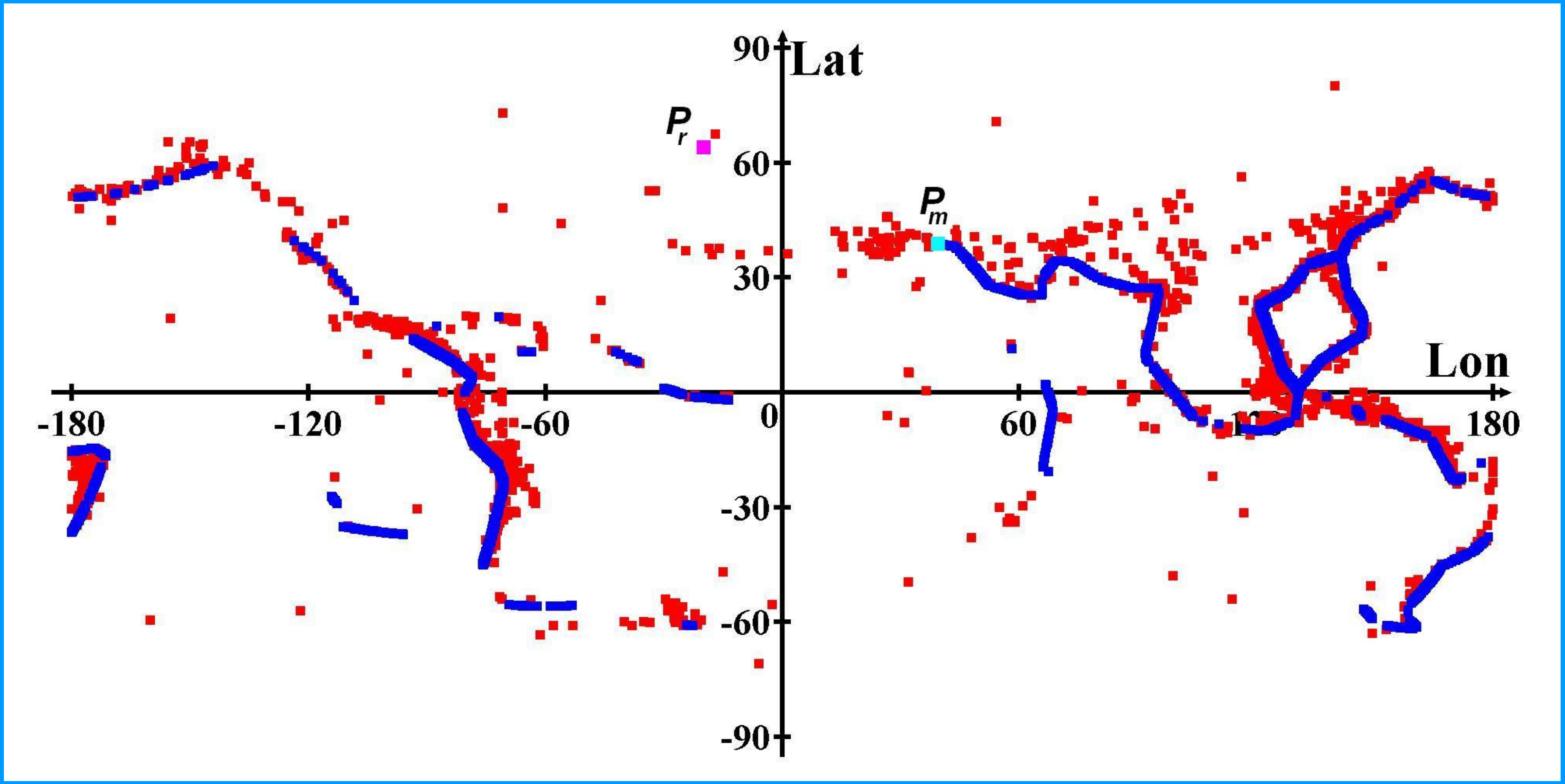
Pros:

- Two main seismic belts, the Circum-Pacific and Alpine-Himalayan (the first is more pronounced), where most of the strong earthquakes occur.
- Increased seismic activity associated with triple junctions of plate boundaries.
- The strongest events in the model occur at the same plate boundaries as in reality.

Cons:

- The absence of events outside fauls zones, inside plates.
- The absence of events on some, obviously active, faults.
- The difficulties of determination/confirmation of some patterns typical for real seismicity. The danger of data-fitting.

HAUSDORFF DISTANCE (between real and model events)



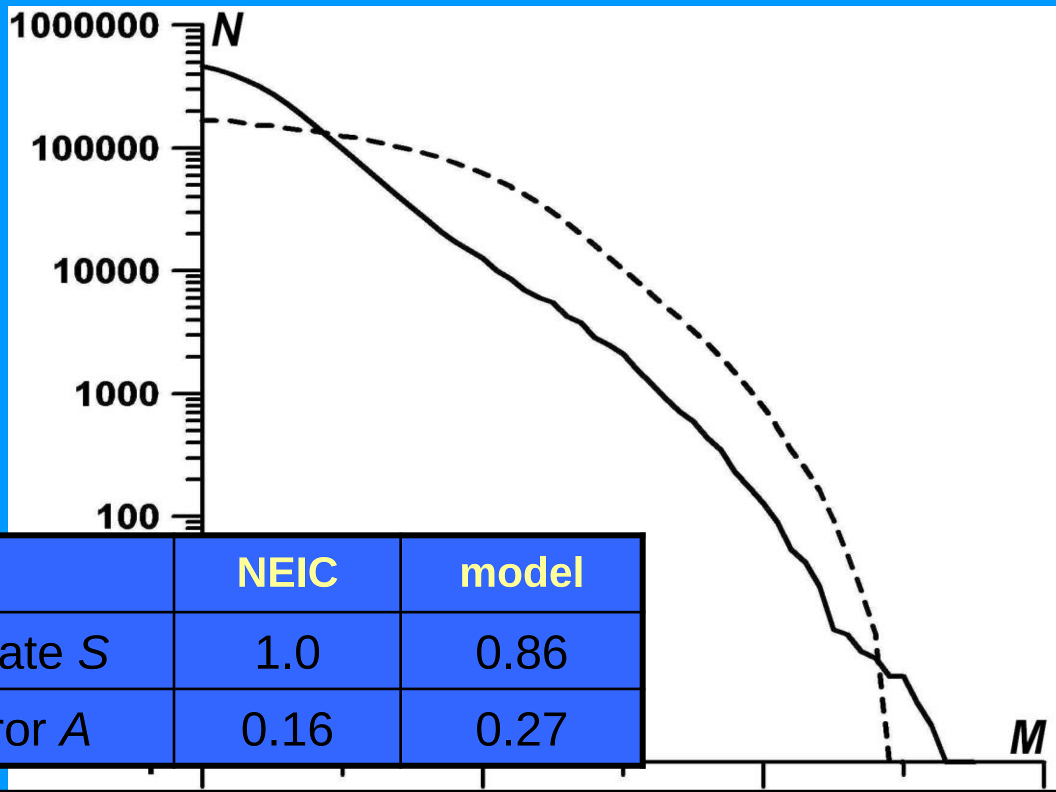
Two sets of epecenters: real events are marked by **red** points, model, by **blue**. The points providing the distance are noted as P_r *and* P_m.

DISTRIBUTION OF EVENTS IN DEPTH (real and synthetic catalogs)

depth	NEIC	model
up to 10 km	0.16	0.14
[10, 40 km]	0.47	0.65
over 40 km	0.37	0.21

In shares with respect to the total number of events with magnitude ≥ 4.0.

REAL / MODEL FREQUENCY-MAGNITUDE PLOTS



	NEIC	model
slope estimate *S*	1.0	0.86
approx. error *A*	0.16	0.27

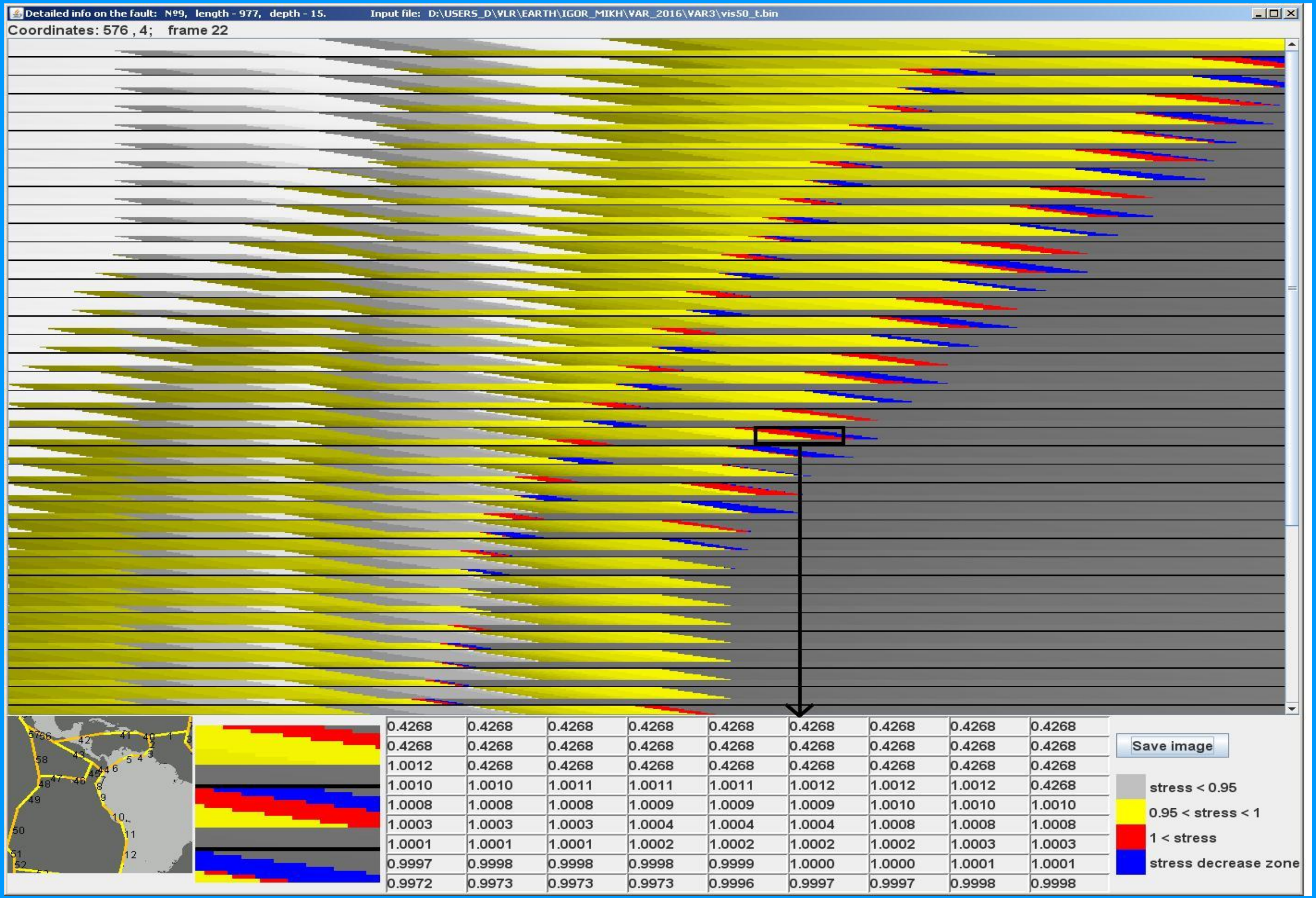
Approximation: linear regression $lgN = c - SM$, LSM.

Error: average distance between real points and LSM-line.

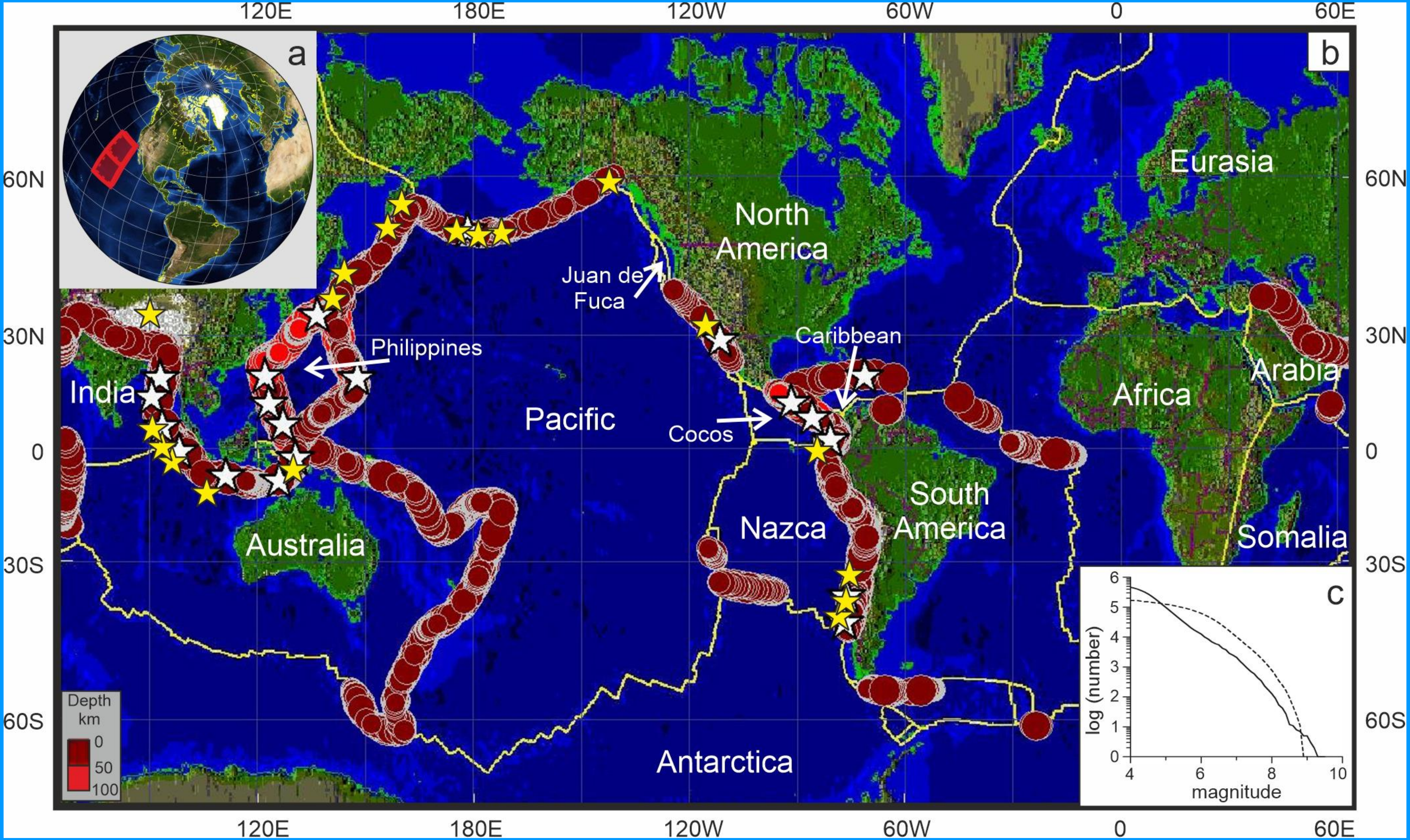
Model variant: magnitude interval [5.0, 8.5].

NEIC: magnitude interval [4.0, 9.0].

MIGRATION OF MODEL EVENTS ALONG TECTONIC FAULT



RESULTS: REAL DATA vs MODEL DATA



INTERMEDIATE CONSLUSIONS

- some model equations taking into account the influence of random factors are modified;
- a draft optimization procedure automating the process of calibrating model parameters is developed on the base of a special quality criterion;
- series of numerical experiments are performed;
- approaches to a detailed verification of simulation results with minimal human expertise in order to consider the possible usage of the spherical block model in a multi-criterion system of seismic risk monitoring.

THANK YOU
FOR ATTENTION!