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THE SHEAR STRESS OF DEEP EARTHQUAKES

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In the studies of deep earthquakes it is generally assumed that they are caused by a volume collapse due to a phase change in a portion of the Earth mantle limited by an appropriate surface. The variation of the volume in turn generates stress changes in the surrounding part of the mantle. We here assume that the surface may be approximated by an ellipsoid of revolution and study the maximum shear stress (*mss*) caused by a uniform normal stress applied to the surface of the ellipsoid in order to establish a link between the shear waves observed after the deep earthquake and the shear stress drop in the medium around the surface of the collapsing body. The seismic moment due to the collapse is then estimated; one approach is based on an estimate of the stress drop occurring in a volume around the collapsed body, the other is based on an estimate of the displacement occurring on the surface which contained the collapsed body.

ТАНГЕНЦИАЛЬНЫЕ НАПРЯЖЕНИЯ ПРИ ГЛУБОКИХ ЗЕМЛЕТРЯСЕНИЯХ

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При исследованиях глубокофокусных землетрясений обычно принимается, что они возникают от объемного разрушения, вызванного фазовым переходом в некоторой части мантии Земли, ограниченной соответствующей поверхностью. В свою очередь, это изменение объема вызывает изменение напряжений в окружающем объеме мантии. Мы принимаем, что эту поверхность можно аппроксимировать эллипсоидом вращения и изучать максимальное тангенциальное напряжение, вызванное однородным нормальным напряжением, приложенным к поверхности эллипсоида для того, чтобы установить связь между поперечными сейсмическими волнами, регистрируемыми после глубокого землетрясения и падением тангенциальных напряжений в среде вокруг поверхности обрушенного объема пород. После этого оценивается сейсмический момент обрушения. Один из подходов к такому оцениванию основан на падении напряжения, определенном в некотором объеме вокруг обрушенного тела, а другой использует оценку смещения, происходящего на поверхности, содержащей обрушенное тело.

Introduction

The recording of earthquakes at a depth of several hundred kilometers was a great surprise in seismology, but the studies of the cause of the earthquakes lead readily to the phase changes with associated volume contractions. Its was also a surprise when the studies of the focal mechanism of these earthquakes discovered that they have a shear component which, at first sight, should not be caused by volumetric changes. However, a simple computation of the stress field in an infinite medium containing a spherical cavity C subject to a uniform normal stress p shows the existence of a shear component. When the stress p has a change dp, the computations show that it causes a change in shear field which reaches its maximum value $0.75dp$ at the surface of C. The same may be easily extended to a spherical shell whose inner cavity is subject to a uniform normal stress which would be a better, although not yet satisfactory, model of the Earth. However, spherical cavities subject to uniform normal stress could be only a very crude first approximation for the surface limiting the body of rock collapsing in a phase change, a better approximation is the surface of an ellipsoid of revolution with semiaxes which may be selected at will for the computations.

In fact in the case of an Earth model whose physical and chemical properties have spherical symmetry, if the physical conditions required for the phase change to occur are satisfied at a point P, they would also be satisfied at any point of the spherical surface S, centered in the Earth's center, through P. The nucleation in P would then propagate preferentially, and perhaps with grater velocity, along S than in the normal direction to S. The phase change would then occupy a volume which may be tentatively approximated with and ellipsoid of revolution with semi-minor axis limited by the physical conditions necessary for the phase change to occur.

Since the tangential propagation of the phase change $t\nu_t$, with t time and ν_t tangential velocity of propagation, is larger that the radial one $t\nu_r$ and the latter is limited by the physical conditions required for the phase change, the flattening of the ellipsoidal volume is bound to approach unity and the mss at the equator of the volume will increase possibly above the fracture limit.

It is often assumed in seismology that geological faults are geometrically described by ellipsoids of revolution with almost unit flattening (e.g. [8]). The case where the elastic medium with an embedded ellipsoidal cavity is subject to a uniform normal tension or compression is of interest in the geological and volcanological applications, because they represent practical cases of magmatic chambers or faults.

The problem is of interest also to the studies of Earth's deformations, which are often observed in underground cavities, because the cavity modifies the strain field in the vicinity as pointed out by King and Bilham [9] who also suggested that this effect could account for many of the inconsistencies in tidal tilt observations. The practical computations of the displacements were made by Harrison [7] and Sato and Harrison [12] who estimated this effect and found that in fact relevant corrections are needed when the observations of strain are made in cavities where for instance the tilt on the surface of the cavity may have great variations from place to place because, as shown by Caputo and Console [4], in some particular cases, there are points where the tilt may result nil.

The problem of estimating the stress field caused by a cavity in a body subject to stress is also of great interest in applied mechanics and has been treated by Neuber [10, 11] who estimated the displacement and stress fields in the infinite medium when it is subject to a shear parallel to the equator of the cavity or to a stress normal to it.

Keilis Borok [8] used the formulae of Neuber [10, 11], considering an infinite isotropic medium in which the strain becomes uniform at large distances from the cavity, and applied them to obtain the relation giving the displacement of the surface of the cavity as a function of the applied shear in the limiting case where the flattening of the cavity is unity, which is of great interest in the studies of the earthquake source with a double couple. Similar results where obtained also by Eshelby [5].

These authors however have not considered the case where the cavity is subject to surface stress.

Finally Caputo [2] extended a work of Neuber [10, 11] to the case where the surface of the cavity is also subject to forces, specifically a uniform normal stress, in the case where the medium is anelastic, presented formulae which could be specialized to the case where the only force field acting is the uniform normal stress at the surface of the cavity and showed that this stress generated a shear stress field in the surrounding medium. Of particular interest is the study of the stress field around the cavity also because it governs the possible generation of successive fractures.

Caputo and Console [4] computed the displacement field, the maximum shear stress (mss) field and its direction in the cases when the medium and/or the cavity are subject to several types of forces finding also that there is a tubular region around the equator of the ellipsoidal cavity with a large stress concentration and that the concentration factor, in general, is inversely proportional to the maximum radius of curvature at the equator; these authors discussed also the possibility of fractures in this region and their orientations. However also Caputo and Console [4] have not addressed directly the problem of the practical computation of the stress field in the infinite medium where the cavity is simply subject to a uniform normal stress which is of great interest in studies of deep earthquakes in order to establish a link between the observed shear waves and the shear stress drop in the medium in the vicinity of the surface of the collapsing body.

This stress drop may be due to the change of volume caused by the phase change in a portion of the Earth mantle limited by an appropriate surface. The variation of the surface in turn generates stress changes in the surrounding part of the mantle. We then assume that this surface may be approximated by an ellipsoid of revolution and study the *mss* in and around the surface itself. How fast the volume and surface change occurs is not important, the shear stress accumulates and, when a critical value is reached, the earthquake may be triggered. The volume variations associated with the phase change may reach values of about 10% for instance in the case of the change from olivine to spinel [6, 14].

In this note, using the formulae of Caputo [2] and Caputo and Console [4], we therefore compute the stress field in an infinite medium containing an ellipsoidal cavity with the symmetry of revolution where a uniform normal stress is applied. We may also add that even if the process of phase change were to stop when it has reached a given volume, the rheology of the medium, owing to the surrounding pressure, will increase the flattening of the surface limiting the ellipsoidal volume and therefore increase the possibility of a fracture [2].

The computation of the maximum shear stress (mss) field

We consider an infinite elastic medium with a cavity limited by an ellipsoid of revolution and the following system of ellipsoidal coordinate system u, ν, w:

$$\begin{aligned} &x = I \sinh u \cos \nu, \\ &y = I \cosh u \sin \nu \cos w, \\ &z = I \cosh u \sin \nu \sin w, \\ &(z^2 + y^2)/\cosh^2 u + x^2/\sinh^2 u = I^2 \end{aligned} \tag{1}$$

with the first fundamental form

$$\begin{aligned} ds^2 &= h^2(du^2 + d\nu^2) + h_w^2 dw^2; \\ h^2 &= I^2(\cosh^2 u - \sin^2 \nu); \quad h_w = I \cosh u \sin \nu \end{aligned} \tag{2}$$

The surface coordinates $u = \text{const}$ are ellipsoids of revolution with major and minor semiaxes $I \cosh u$ and $I \sinh u$ respectively, the surfaces $\nu = \text{const}$ are hyperbolas, both have revolution symmetry about the x-axis, the surfaces $w = \text{const}$ are planes through the x-axis.

We assume that the ellipsoidal cavity is defined by $u = u_0 = \text{const}$.

The parameter of interest in the present discussion is u_0 which defines the ratio f of the semiaxes of the ellipsoid

$$f = \tanh u_0. \tag{3}$$

When u_0 is nil, f is nil and the cavity becomes a flat disk; when u_0 is infinite, then f is unity and the cavity becomes a sphere. It is worth noting that the flattening of the ellipsoid is $1 - f$. Since only the relative dimensions of the semiaxes of the ellipsoid are of interest we will assume $I = 1$ in which case, changing u_0, both semiaxes of the cavity change length.

We computed the stress components σ_u, σ_ν, σ_w, $\tau_{u\nu}$ and the maximum shear stress (mss) at the equator and at the pole of the cavity. The value of mss is minimum at the pole and maximum at the equator where the curvature of the cavity is maximum. In the plane of the equator outside the cavity the mss decreases with distance. The formulae expressing the stress components are those obtained by Caputo [2] and used already by Caputo and Console [4]. The mss at the equator of the cavity is shown in Fig. 1 as a function of the ratio f of the semiaxes. One may see analytically that when f is nil the mss is infinite; in the figure one may also note that the mss is a decreasing function of f.

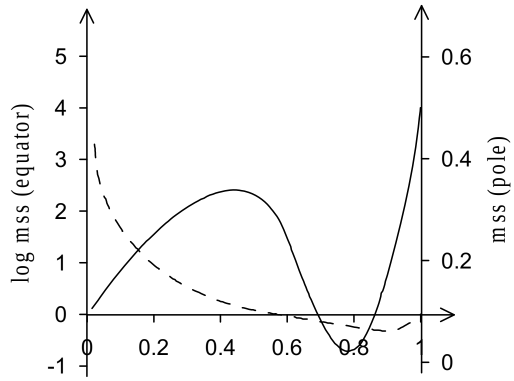

Fig. 1. Maximum shear stress (*mss*) at the surface of ellipsoids of revolution with minor and major semiaxes b and a respectively, as a function of the ratio b/a.
The solid curve gives the *mss* at the pole of the ellipsoids with scale to the right, the dashed curve gives the *mss* at the equator of the ellipsoids with scale to the left. The mss is measured in units of the normal stress supplied to the surface of the ellipsoid

A practical form of the collapsed body could be that defined by $f = 0.01$ in this case, the major semiaxis of the ellipsoid is 100 times larger than the minor semiaxis or the body is almost a lens limited between two close Earth's radii; then it is seen from the Fig. 1 that the *mss* at the equator is more than 100 times the normal stress applied to the surface of the cavity. This enormous concentration of stress at the equator of the cavity could be the nucleation of fracture causing the earthquake and explaining the presence of shear in the focal mechanism of deep earthquakes; neglecting the possible presence of a local tectonic stress, the stress drop of an earthquake is limited by the *mss* at the equator of the cavity and could be used as an upper limit for the *mss* caused by the phase change.

Conclusions

Concerning the seismic moment we will consider two tentative approaches for its estimate. One approach (a) is based on an estimate of the stress drop occurring in a volume surrounding the collapsed body, the other (b) is based on an estimate of the displacement occurring at the surface which contained the collapsed body.

It is assumed in case (a) that the stress drop is of the order of the strength of rocks which in turn increases with the confining pressure [13]. A possible depth of a deep earthquake may be taken 300 km where we assume a strength of 2000 MPa extrapolating the data shown by Scholz [13] which seem to level off at a confining pressure of 800 MPa with a value of 2000 MPa.

If the ellipsoidal volume of the collapsed body has major semiaxis l, minor semiaxis fl, and stress drop p, assuming that, due to the concentration of stress at the equator of the ellipsoidal volume, the stress drop is that of fracture about 2000 Mpa and involves a volume surrounding the collapsed body as large as that of the body, the seismic moment is

$$M_0 = (4\pi/3) f l^3 p, \tag{4}$$

whose values are shown in Fig. 2, with $f = 0.01$ and l in the range [1,50] km.

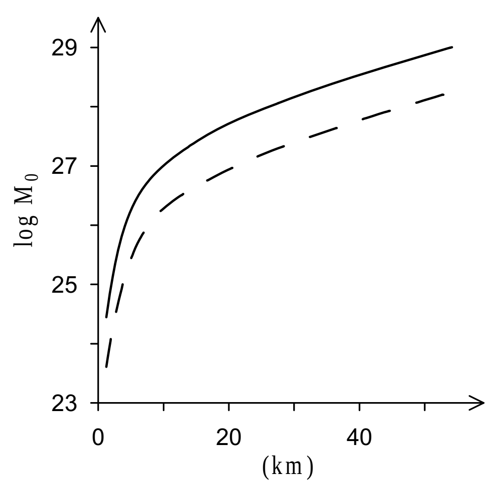

Fig. 2. Estimates of the seismic moment as a function of the radius of the equator of the ellipsoidal collapsing body under the hypothesis of a volume change in the final phase of the collapse (dashed curve) and of a stress drop in the surrounding volume in the final phase (solid curve)

In the hypothesis (b) we tentatively assume that the displacement on the upper and lower surface of the cavity containing the collapsed volume is about 0.001 of the minor semiaxis, that is, $0.001fl$, which corresponds, with the assumed flattening close to unity ($f = 0.01$), to a volume change in the final phase of the collapse of 0.002%, the seismic moment is [2]

$$M_0 = \pi 0.001 f l^3 (\lambda + 2\mu/3) \tag{5}$$

where we put $\lambda = 2\mu = 2 \times 10^{12}$. The values of (5) are presented in Fig. 2 with $f = 0.01$ with l in the range [1,50] km. The difference with the values of the hypothesis (a) is one unit of $\log M_0$.

The partial agreement of the values given by formulae (4) and (5) is only fortuitous since the value of the stress drop used in formula (4) is extrapolated from the values observed to 800 MPa and the value of the displacement used in formula (5) is an assumption and is not observed.

There would be several possibilities to explain the different results, the most obvious is that both, displacement and stress drop are crude estimates, also the formulae used for the seismic moment are approximations only and, finally, it is possible that the linear theory is not adequate.

As a final comment we note that, since the stress is released in a cavity, shear will be released and shear waves will be generated, which was surprising in the studies of deep earthquakes.

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