IV. СЕЙСМИЧНОСТЬ И МОДЕЛИ

УДК 550.34

THE PLACE OF THE GUTENBERG–RICHTER LAW AMONG OTHER STATISTICAL LAWS OF NATURE

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The paper is a presentation of results published by the author since 1970 and those yet unpublished in attempts to develop a unified view, or method of consideration, for natural phenomena and their statistics. In many cases we know the forces and/or forcings, i.e. the power input into the systems in question. The systems can be in steady states due to various kinds of dissipative processes, or events releasing accumulated energy. It is noted that the velocity, or energy, is proportional to the force, or the forcing, times the time determined by either the external factors, or internal ones for non-linear phenomena. In statistics of events we measure their energy and if we know their forcing we can estimate their waiting times, or frequency of events. The size of an earthquake, EQ, is measured by its seismic moment, M. Kagan estimated in 1997 the global rate of generation of M which is shown here to be thrice the global geothermal heat flux, the ultimate source of all geodynamical processes. The Gutenberg-Richter law is explained in these general terms and many statistical relationships for EQ are checked, tested and found. Only a small fraction of a per cent of the global heat flux goes into generation of EQ energy. The induced EQ are considered and forcing for them is determined as $V dp/dt$ where p is the pressure deviation from isostasy and V is the volume where such deviations are observed. The statistics for tsunami and landslides are explained in similar terms and found to be close analogs for EQ. More complicated analogs are energy spectra for cosmic rays. For all these phenomena the energies are the forcing times the cumulative waiting times. This is characteristic of processes with delta-correlated stochastic forces which is also valid for stochastic flows in hydrodynamics. The times here are determined as minimal ones inherent in the system and can be found from the similarity criteria. The last should be presented as ratios of two times and the smaller one is used to estimate the mean flow velocity, or energy, of course, up to a numerical factor. The Kolmogorov turbulence theory results, equilibrium range for sea surface wave spectra, winds in planetary atmospheres, various regimes of convection, including ones with rotation, flows in pipes, etc. are considered in this way, all forming a family tree with Gutenberg-Richter law being a branch of the tree.

МЕСТО ЗАКОНА ГУТЕНБЕРГА–РИХТЕРА СРЕДИ ДРУГИХ СТАТИСТИЧЕСКИХ ЗАКОНОВ ПРИРОДЫ

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В статье представлены результаты, публикуемые автором с 1970 г. и еще им не опубликованные, в попытках развить единый метод рассмотрения природных явлений и их статистических свойств. Во многих случаях мы знаем силы и/или воздействия, т.е. мощность, поступающую в рассматриваемую систему. Система способна быть в некотором среднем по энергии состоянии, благодаря присущим ей различным диссипативным процессам, или событиям, сбрасывающим аккумулируемую энергию. Отмечалось, что скорость или энергия пропорциональны силе или воздействию (мощности), умноженным на время, определяемое либо внешними, либо внутренними факторами в случае нелинейных явлений. В статистике событий мы измеряем их энергию, и если знаем воздействие, то можем оценить их времена ожидания (кумулятивные), т.е. частоты событий. Величина землетрясения (ЗТ) измеряется его сейсмическим моментом M. Каган оценил в 1997 г. глобальную скорость генерации величины M, которая, как показано здесь, равна утроенному значению глобального геотермического потока тепла, изначального источника всех геодинамических процессов. Закон Гутенберга–Рихтера просто объясняется из этих общих представлений, и многие статистические закономерности для ЗТ проверяются, тестируются и отыскиваются. Лишь небольшая доля процента глобального потока тепла расходуется на генерацию энергии ЗТ. Рассмотрены индуцированные ЗТ, и мощность (воздействие) для них определена как $V\,dp/dt$, где p – отклонение давления от изостазии и V – объем, в котором эти отклонения наблюдаются. В подобных же терминах объяснены статистики для цунами и оползней, законы распределений для которых найдены близкими аналогами таковых для ЗТ. Более сложными аналогами являются спектры энергии космических лучей. Для всех этих процессов энергия есть произведение мощности на время ожидания. Это характерно для процессов с дельта–коррелированными случайными силами, что также верно для стохастических течений в гидродинамике. Здесь времена определяются как минимальные, присущие системе, и могут быть найдены из критериев подобия. Последние должны быть представлены как отношения двух времен, и наименьшее из них используется для оценки средней скорости потока или энергии, конечно, с точностью до численного коэффициента. Результаты для теории турбулентности Колмогорова, равновесного интервала частотного спектра морского волнения, ветров в атмосферах планет, различных режимов конвекции, включая вращающиеся жидкости, течений в трубах и т.п. рассмотрены подобным путем и все они формируют семейное дерево, для которого закон Гутенберга–Рихтера является одной из ветвей.

Introduction

Many natural regularities have statistical nature and often are considered as purely empirical ones. The scatter of the data relative to the proposed relationships is usually rather large and this is an extra argument to assume an absence of some general principles determining the broad outline of the process behavior. One can easily relate to this category the Gutenberg-Richter, G.–R., law describing the frequency-size distribution of earthquakes. It is usually written in the form

$$\lg N = a - bm, \tag{1}$$

where N is the number of EQs in a certain region, or on the globe, during certain time period, usually mean per year with magnitude equal or greater than m; a is a value depending on the region and on the period, b is the numerical coefficient close to unity (0.9 for the Southern California [1]), m is the earthquake magnitude determined from observations by the procedures described in all textbooks on seismology.

Since Gutenberg time it was accepted that for very large EQs the value of b is close to 1.5 [2]. Similar result has been established for EQs occurring near the mid–oceanic ridges, the places of the new earth's crust formation [3].

During last decades in the scientific literature for the characterization of the EQ size the value of seismic tensor is used whose scalar value is determined as (e.g. [4])

$$M = \mu S u \tag{2}$$

where μ is the shear modulus, S is the area of the crust fracture, u is the mean slip in the EQ, i.e. the mean displacement of the blocks adjacent to the fracture. The scalar value of the seismic moment is related to the magnitude m by the (statistical in origin) relationship

$$m = \frac{2}{3}\lg M - 6 \tag{3}$$

where M is in SI units: $N \cdot m$.

In the future text we shall use one more formula by Richter relating the magnitude m to the energy of the emitted seismic waves E (Joules).

$$\lg E = 1.5m + 4.8$$

From here and from (3) we can relate E to the seismic moment M as

$$\lg E = 1.5M - 4.2,$$

or in other terms:

$$E = 6.3 \times 10^{-5} M \tag{4}$$

all in SI units. It is possible, because formally the units for the moment $N \cdot m$ are the same as the units for work, or energy: Joules.

In these new variables the G–R law can be written as:

$$N = AM^{-2b/3} \tag{5}$$

where N is the frequency of EQs, here in Herz, and the value of A is

$$A = 10^{a+6b} \tag{6}$$

Originally the idea on the similarity of EQs belongs to Aki [5]. Later on the idea was used and developed by many researchers from which we mention [6–10]. All of them obtained $b = 1$. They supposed, in some accord with observational data, that certain EQ characteristics do not depend on the size of EQs: the stress drop at the fracture, the ratio of the fracture length L to its width W, etc. We shall check especially a degree of validity of the two assumptions later on in this paper (see Figs. 1 and 2).

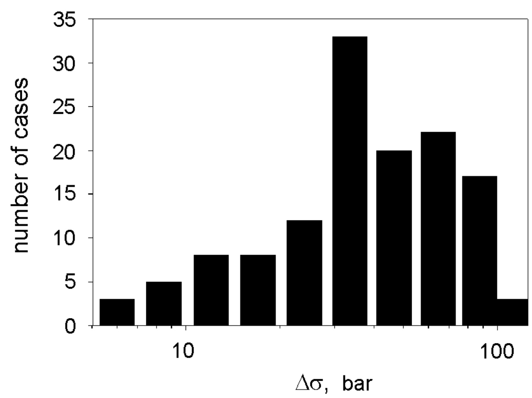

Fig. 1. Histogram of the stress drop values

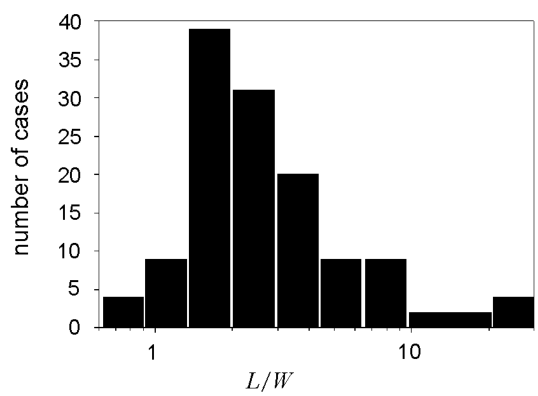

Fig. 2. Histogram of the length to width ratio

However all above mentioned papers did not give a clear physical definition of the nature of the values of A in (6), or a in (1), though a semiempirical determination was proposed by Rundle [10]. Such a definition was given by the author [11] using methods of the similarity theory. H. Jeffreys have also proposed an explanation of the nature of more than the century old Omori law or the frequency-size distribution of aftershocks. For the lack of space, 9 did not formulated a number of essentiall details in my 1996 paper. For the past 5 years important applications of the results described in 1996 have been used for explaining the induced (man-made) EQs [12] and to starquakes [13]. It was also shown [14, 15] that EQs and their statistics are determined by a sufficiently general rule ("principle") of the fastest response of a system on an external forcing. This paper discusses all these questions. Though it is largely a review it also contains a number of new results related to EQs and also to the frequency spectra in systems with random external forcings (turbulence, sea surface waves, stresses within the earth's crust, etc.). It is also shown how the above mentioned "principle", or rule, originates from the Newton's second law of mechanics.

1. The similarity theory for earthquakes

Most natural and logically grounded is the use of the similarity theory in problems rigorously posed mathematically [16–18] where the problem is described by a system of equations with initial and/or boundary conditions. The equations and conditions contain normally dimensional parameters external to the system from which one can build up the scales of length, time, of other problem variables. Using these scales one can put down a system with its conditions in a dimensionless from. The dimensionless parameters originating in this process characterize regimes of the processes in question. Often the values of the non-dimensional parameters are very large, or very small, giving one a possibility to obtain non-trivial results in a finite form, though the situation may be more complicated (see especially [17, 18]) when one could not neglect very large, or very small, parameters of similarity, i.e. not expect the self-similarity in respect to precise value of large or small quantities entering the parameters. The best known example is the flow of a viscous fluid when the size of the flow, r, and its velocity, v, at an external flow boundary are known. Then the ratio of dynamical non-linear advection forces to the viscous forces is characterized by the Reynolds number:

$$\mathrm{Re} = vr/\nu \tag{7}$$

where ν is the kinematic viscosity of the fluid. At $\mathrm{Re} \leq \mathrm{Re}_{cr} = O(10^3)$ the flow is laminar but for $\mathrm{Re} > \mathrm{Re}_{cr}$ it is turbulent. Unfortunately, the seismology is still "in the pre–equation state" [19], i.e. the problem of stress accumulation in the crust, mainly due to plate tectonics, and the formation of fractures there releasing excess stress, is not yet mathematically formulated due to excessive complexity and multiplicity of all processes involved.

However, as the practice of science shows, if one has a rational understanding of the mechanics and physics of processes in question it is possible to obtain some useful results even without complete mathematical description of the processes. Doing this one should choose determining (dimensional) quantities characterizing the medium, as well as the process itself. The medium here, the brittle crust, is characterized by the thickness of the crust h, by its shear modulus μ with the dimension of pressure, or energy per unit volume. We shall further use dimensions of energy, e, length l, and time t. These units are more natural for the problem in study. According to the data of about 200 EQs described by Purcaru and Berkhemer [20] (further on PB), the value of the shear modulus μ change rather little and is between 2.8 and 7 by 10^{10} $\mathrm{N\,m^{-2}}$ or Pa, with an averaged value about 4×10^{10} Pa (Pascals). Further on we consider the value of μ as constant.

Astonishingly small is the variability of the stress $\Delta\sigma$ released at the quakes. This has been known for quite a time and was used in papers mentioned in the introduction to understand the basic regularities of the quakes. We especially checked a degree of constancy of the released stress and the results are presented as the histogram at Fig. 1. It was constructed using data for 131 EQ found in PB for depths less than 70 km plus 5 additional quakes: Lisbon 1755, New Madrid 1811, Charlston 1886, and two Kuril 1994 found in the literature. Each bin at Fig. 1 differs from the next one by a factor $10^{0.14} = 1.38$. The median value is 41 Bar = 4.1 MPa. The scatter is large because the values of $\Delta\sigma$ found in PB lie between 5 and 125 Bar. But if one recalls that the values of the seismic moment M used here are from 1.2×10^{-3} to 2400 in 10^{20} N m, i.e. differing by a factor 2×10^6, one can accept that the relative variability is not large, especially noting that there is no evidence on the dependence of $\Delta\sigma$ on M.

Therefore as a rough estimate one can use the value $\Delta\sigma \approx 4$ MPa as a representative constant value. This assumption on $\Delta\sigma \approx \mathrm{const}$ was a base for introducing the notion of the self–similarity of EQs in the references cited in the Introduction. It might be considered as a justification for invoking the notion SOC, self–organized criticality, into the EQ studies [21].

The other value sometimes used as another base for the self–similarity consideration of EQs is the constancy of the ratio L/W, fault length to its width. We checked this assumption using all data in PB having their values plus 5 aforementioned large quakes. The results are presented as a histogram at Fig. 2 for 163 events. This ratio was found to vary between 0.5 and 30, considerably larger relative to changes in the stress drop. We divided the range of variation into 11 bins, subsequent bins differing by the factor $10^{0.17}$=1.48. The median value of L/W was found to be 3.7. The scatter here is 2.4 times larger than for the values of $\Delta\sigma$ but we shall not consider this ratio in the discussion. From geometrical characteristics we shall use the length area of faults and the slip at the rupture between adjacent blocks.

The most important point in considering the process of EQs formation is the quantitative characterization of the stress growth in the crust, i.e. increase of elastic energy of deformation. It is now a common view that this increase is due to the plate tectonics caused by the mantle convection. Because of that the main number of EQs is found near the boundaries of lithospheric plates. The power source for the global process of the mantle convection is the geothermal heat flux. Therefore purely local consideration of the processes of preparation and realization of the EQs is evidently not sufficient. Also strong EQs for concrete seismically dangerous zones are, fortunately, rare on the time scales of a person's life time or during the work time of the modern seismological network. Therefore we shall use the global catalogues and, as the consequence, consider the total geothermal flux, as an ultimate power source for EQs. Its value is $F = 4 \times 10^{13}$ W if one assumes its mean density to be $0.08\ \mathrm{W\,m^{-2}}$. Such an approach was taken by Golitsyn [11]. A year later Kagan [22] gave an estimate directly of the global seismic moment value growth as $\dot{M}_0 = dM_0/dt = 36 \times 10^{20}\ \mathrm{N\,m\,yr^{-1}}$, which gives $\dot{M}_0 = 12 \times 10^{13}\ \mathrm{N\,m\,s^{-1}}$. Because formally Newton times meter per second = Watt, we may write

$$\dot{M}_0 \approx 3F$$

Also due to the relationship between seismic wave energy and the seismic moment(4) we can estimate the power of these waves on the global scale as

$$\dot{E}_0 = 6.3 \times 10^{-5} \dot{M}_0 = 1.6 \times 10^9\ \mathrm{W} \tag{8}$$

We see that only 1.9×10^{-4} of the geothermal flux is spent on the seismic waves generation.

We may hope now to list all important dimensional parameters: the shear modulus μ and the stress drop $\Delta\sigma$, both with dimensions of pressure, or energy per unit volume, the seismic moment M with dimension of energy, and geothermal flux F, or seismic moment generation rate $\dot{M}$, with dimension of power.

We can form various scales now. Consider first the time scale

$$\tau(\geq M) = M/\dot{M}_0 \approx M/3F \tag{9}$$

We represent here the cumulative time of an event with the moment equal or larger than M. The differential waiting time $\tau(M)$ of an event characterized by the value of M within the interval $M - \Delta M \leq M \leq M + \Delta M$ can be defined as follows

$$1/\tau(\geq M) = \int_M^{+\infty} 1/\tau(M)dM$$

The scale of length determined as

$$L(M) = (M/\Delta\sigma)^{1/3} \tag{10}$$

which corresponds to the "EQ volume", a notion introduced by Tsuboi almost half a century ago (see [23]):

$$V = M/\Delta\sigma = L^3(M). \tag{11}$$

The length scale $L(M)$ from eq.(10) is a good estimate of the fault length and, especially, its area. Fig. 3 presents the scatter plot of data from PB for dependence $L(M) = a(M/\Delta\sigma)^{1/3}$, with $a \approx 2$ and the determination coefficient $r^2 = 0.869$. Fig. 4 gives the similar dependence for the fracture area

$$S(M) = b(M/\Delta\sigma)^{2/3} \tag{12}$$

with $b \approx 2$. Note that using eqs. (10) and (12) for theoretical estimates of the fault length and area

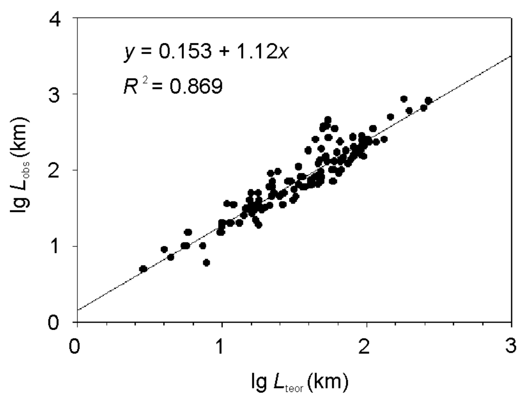

Fig. 3. The scatterplot of the observed rupture length against the theoretical value by eq. (10)

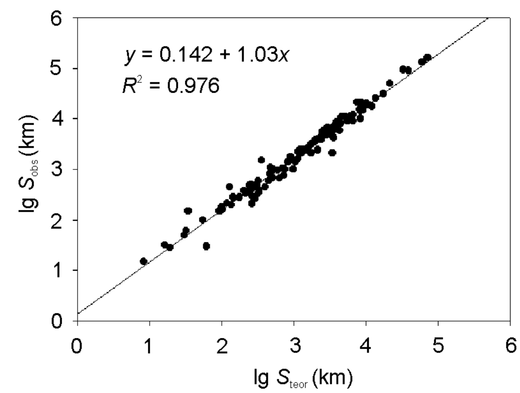

Fig. 4. The scatterplot of observed rupture areas against the theoretical values by eq. (12)

we are using observed values of $\Delta\sigma$, not just 4 MPa. This noticeably reduces the scatter of points.

Recalling eq.(3) relating the magnitude m of an EQ with its seismic moment one can see the direct relation between m and $\lg S$. So, not considering theory one can build up a scatter plot of m against observed area S, again using PB data. It is shown on Fig. 5. The magnitude m varies between 5.2 and 8.7 and the area varies over 4 orders of magnitude. The straight line is

$$m = \lg S(\mathrm{km}^2) - 4.0 \tag{13}$$

with the determination coefficient $r^2 \approx 0.832$. If we would measure the fault area in ars (1 ar= 100 $\mathrm{m}^2 = 10^{-2}$ hectar $= 10^{-4}\mathrm{km}^2$), then the relationship would have especially simple form:

$$m = \lg S(\mathrm{ar}) \tag{14}$$

Similar relationship one could find in Kasahara (1981) for Japanese data.

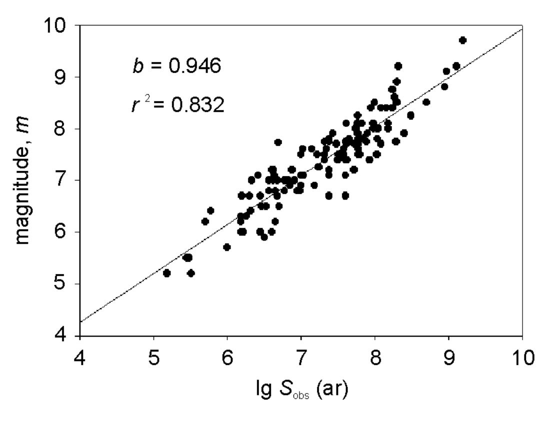

Fig. 5. The scatterplot of the earthquake magnitude against the observed area

So, the Gutenberg-Richter magnitude m is a good estimate of the decimal logarithm of the fault area. The value of S can be estimated up to a factor of 3 if one knows the EQ magnitude as can be seen from Fig. 5. A relationship similar to (13) was also obtained by Kanamori and Anderson [6] assuming the constant value of L/W, only instead of 4.0 there they had 4.1 taking into account considerably smaller number of EQ. The relationship (14) suggests together with eq. (1) that the cumulative number of EQ is inversely proportional to the earthquake fault area. That means that the rupture area S is a natural measure of the EQ intensity with magnitude m being the decimal logarithm of the area measured in ars.

As a belated example of eq. (14) performance let us consider the Antarctic earthquake, March 25, 1998. It is described in a series of papers in Geophysical Research Letters, v. 27, No. 15, August 1, 2000. Its magnitude in one paper is 8.1, in another 8.2, the length 178 ± 46 km and the width 80 ± 53 km. With the mean area 178×80 km^2 $= 1.424 \times 10^8$ ar we have from (14) m=8.15. Of course, such a coincidence may be, considered as a fortuitous, but the magnitude would be $m = 8.47$ for the maximum area, and $m = 7.55$ for minimum case. The seismic moment is estimated there as $M = 2.8 \times 10^{21}$ N m, and the shear modulus as 3.2×10^{10} Pa. Then from (2) we have the mean slip 6.1 m. Figs. 3 and 4 determine a statistical dependence of the fault length, L, and area, S on the length scale $L_\phi = (M/\Delta\sigma)^{1/3}$. Both have numerical coefficients about 2. Therefore we may use these results for estimates of the stress drop $\Delta\sigma$. Using the area $S = 1.424 \times 10^{10}$ m^2 and the seismic moment value we find $\Delta\sigma$ as 4.64 MPa. Therefore the stress drop is 43 ± 3Bar if we take into account uncertainties in the area estimate. All parameters of this Antarctic quake seem to be quite reasonable in terms of our knowledge about EQ and agrees well with a wealth of statistical evidence found by our analysis.

Our choice of external parameters allows one to form 3 non-dimensional similarity numbers. One relates to the material of rocks and it is the Poisson coefficient $\sigma = (2K - \mu)/(3K + 2\mu)$, where K is the bulk modulus. It varies little and normally it is not considered in theoretical seismology. The other is the ratio of the stress drop to the shear modulus

$$\Pi_\mu = \Delta\sigma/\mu.$$

Our analysis of PB data for 136 EQs with depths less than 70 km shows that $\Pi_\mu = (1.13 \pm 0.72) \times 10^{-4}$. We may hypothesize that in the first approximation the exact value of Π_μ is not essential not only because it is small but mainly due to its relatively low variation from an event to event. This hypothesis requires an observational check (otherwise see the warnings of Barenblatt [17, 18] on the self-similarity of the second kind.

The most important similarity criterion seems to be the ratio of our length scale (10) to the brittle crust thickness h:

$$\Pi_h = L(M)/h = (M/\Delta\sigma)^{1/3} h^{-1} \tag{15}$$

The importance of the crust thickness for interpretation of seismicity statistics was noted by Keilis-Borok in 1971 (see [24]): for the frequency-size distribution of earthquakes it was demonstrated by Grigorian [9] and Rundle [10]. Pacheco et al. [2] found that the frequency-size distribution for very large EQs falls more steeply than for the rest having $b \approx 1.5$. Rundle has explained that very strong quakes tear up the hall crust and the fault develops in the one direction whereas for other quakes the fault propagates two-dimensionally along its plane within the crust. He noted that the frequency-size distribution of volcanic EQ may also have $b \approx 1.5$ because they occur in thin layers. The author is not aware of data on volcanic quakes, however the steeper frequency-size distribution was observed near the mid-oceanic ridges generating the crust [3], where it is much thinner than elsewhere. All this suggests that the similarity criterion Π_h is the main one determining the character of an EQ.

Kagan [4] has generalized the G-R law in the form of (5) adding at its r.h.s. a factor $\exp(-M/M_l)$ which makes the moments of the probability distribution finite. He has estimated the limiting value of the moment M_l as 1.6×10^{20} N m. For the South California where the crust thickness is about 25 km we obtain Π_h=1.36=$O(1)$. Note that earlier estimates used $\Delta\sigma = 4.4$ Ma [11] and then $\Pi_h \approx$1.31. One may interpret value of Π_h slightly larger than 1 as a deviation of the fault plane from the vertical. For Π_h=1.36 the angle between the vertical and the plane can be estimated as $\theta = \cos^{-1}(\Pi_h^{-1}) \approx 43°$. The distribution of the fault planes over angles θ could, possibly, be of certain interest.

2. The Gutenberg-Richter law for the earthquakes frequency–size distribution

Let us first repeat with comments the arguments by Golitsyn [11] leading to the G-R law which used only the notions of similarity and dimensional analysis. These arguments lead to the relationship

$$\dot{N}(\geq M) = cF/M \tag{16}$$

where F is the geothermal flux and c is the dimensionless coefficient which may depend on the similarity criterion Π_h. The dependence (16) is already close to the result indicated by Okal and Romanowicz [3] who found for over 800 mid-oceanic quakes $\dot{N} \propto M^{-n}$ with n=1.05. So we see that the simplest formula (16) with n=1 is already catching the dependence on M for $\Pi_h > 1.4$. The coefficient c was found in 1996 to be close to 0.4 comparing (16) with the Harvard EQ catalogues. For oceanic crust 3 km thick the equations (5) and (15) allow us to estimate that $\Pi_h \approx O(1)$ corresponds to the EQ magnitude $m \approx 5.5$, a rather modest EQ size in other standards.

The overwhelming majority of the registered EQs are for $\Pi_h \ll 1$. One has to assume for them that the coefficient $c = c(\Pi_h)$. In 1996 I proposed that we can expand this dependence into the Taylor series for $\Pi_h \ll 1$ and take only the first linear term in the expansion as $c(\Pi) = c_1\Pi$. Then

$$\dot{N}(\geq M) = c_1\Pi_h F M^{-1} = c_1 F M^{-2/3} h^{-1} (\Delta\sigma)^{-1/3} \tag{17}$$

Seismologists are well aware that the coefficient b is close to 1 in (1), therefore the exponent n should be close to 2/3. Indeed Okal and Romanowicz [3] found it between 0.67 and 0.7, Kagan [22] gives $n = 0.64$, Russian geophysicist Smirnov and Ispolinova [25] produced $n = 0.65 \pm 0.02$. Moreover, data of a very sensitive and dense network of seismometers in the South Iceland which is registering EQ down to magnitude $m = 0$, or even smaller, does produce the value of $b \approx 1$ [26]. Using (3) we find that $m = 0$ corresponds to $M = 10^9$ N m. This means that G-R law with the exponent $n \approx 2/3$ is observed over about twelve decades of the seismic moment change, at least for Iceland.

Equation (17) allows us to determine the global value of the term a in (1) as

$$a = \lg[c_1 F h^{-1} (\Delta\sigma)^{-1/3}]$$

The coefficient c_1 was found to be about 0.4. Using now the value of $\dot{M}$ the rate of the global seismic moment change instead of the geothermal flux F, we obtain

$$a = \lg[0.13 \dot{M}_0 h^{-1} (\Delta\sigma)^{-1/3}]$$

and by exponentiating it for the value of A in (6) we get

$$A = 1.3 \times 10^4 \dot{M}_0 h^{-1} (\Delta\sigma)^{-1/3}$$

Using observed value of the seismic waves generation rate $\dot{E}$ from (4) instead of $\dot{M}_0$ we obtain

$$A \approx 0.8 \dot{E} h^{-1} (\Delta\sigma)^{-1/3}$$

This is a global value. To get regional estimates we should know the mean crust thickness there and the corresponding value for the seismic waves energy generation rate. All these would require a thorough regional data analysis.

The G-R law can also be written in terms of waiting time:

$$\tau(\geq M) = 2.5 M (\dot{M}_0)^{-1} f_1(\Pi)$$

The dimensionless function $f_1(\Pi)$ equals to $[C(\Pi)]^{-1}$ from (16) and has now the following asymptotes: $f_1(\Pi) = 1$ for $\Pi \gg 1$ and $f_1(\Pi) \propto \Pi^{-1}$ for $\Pi < 1$. This leads to $\tau(\geq M) \propto M^{2/3}$ for small Π and $\tau(\geq M) \propto M$ for large Π. The bigger the size of an EQ, the larger are weighting times for it.

Another derivation of the G-R law for $\Pi_h < 1$ was briefly described in [13, 14]. It was based on the rule of the fastest response of a system on external forcing formulated in [14, 15, 27]. The rule uses an evident formula

$$B \approx (dB/dt)\tau = \dot{B}\tau. \tag{18}$$

The rule says that if a system possesses several time scales formed e.g. by external dimensional quantities of the problem in study then one should choose the minimal time scale (in linear problems that would correspond to the maximal increment in the system). In the statistics of events characterized by intensity B we often know the forcing, i.e. the value of $\dot{B}$, e.g. the geothermal flux. Then the frequency-intensity distribution law can be formulated for the cumulative frequency (see eq. (9)) from (18) as

$$\dot{N}(\geq B) \approx cB^{-1}\dot{B} = cd\ln B/dt, \dot{N} = \tau^{-1}, \tag{19}$$

where c is a numerical coefficient. The G-R law can be obtained from (18) at $\Pi_h < 1$ recalling that the energy of an EQ is related to "the EQ volume" $V = M/\Delta\sigma$, see eq. (11), while the forcing comes from below due to the mantle convection moving the plates and is spread over the whole crust column with the volume estimated as L^2h. Now we relate (18) or (16) at the left to $V = L^3/\Delta\sigma$ and the right to L^2h and obtain

$$\dot{N}(\geq M) = c\frac{L}{h}\dot{M}M^{-1} \propto M^{-2/3} \tag{20}$$

For $\Pi_h > 1$ "the EQ volume" and the "volume" with stress are the same, and the exponent in the repeatability of EQs is just $n = 1$, as is readily seen in (19).

The G-R law for $\Pi_h < 1$ expressed by (17) or (20) together with the eq. (12) for the fault area S allow us to give another interpretation of the law as the frequency-fault area distribution:

$$\dot{N}(\geq S) = A_1 S^{-1}, \quad A_1 \approx 0.2\frac{F}{h\Delta\sigma} = \frac{0.2F}{h\Delta\sigma} \tag{21}$$

Note that the value of A_1 has the dimension l^2t^{-1}, i.e. of a diffusion coefficient. The relationship (21) can be now presented in the form similar to (18):

$$S = \dot{S}\tau(\geq S) \quad \text{with} \quad \dot{S} = dS/dt = A_1,$$

which clarifies the dimension of A_1, though the analogy with the diffusion coefficient is quite formal because we characterize the EQ intensity by its fault area and because the time $\tau(\geq S)$ in the statistics of events is the cumulative waiting time for an event to happen and form the fault area equal or larger than S.

The existence of the G-R law in the form (21) is no wonder at all. Because we have already established the direct relationship between the Gutenberg-Richter magnitude m and the fault area expressed by eqs. (12) or (14), the form of the law as by (21) is just another form of their original law (1) with $b = 1$ (recall that owing to (3) the exponent n in (20) is related to b as $n = 2/3b$.)

3. Man-induced earthquakes and other phenomena

Various kinds of man activity, such as mining, filling up water reservoirs or undersurface cavities by fluids, pumping out oil or gas, all lead to deviations from the isostasy. As a result the stress is increasing in the crust. When the stress reaches some critical value it is released by an earthquake–like process. These EQ are called induced and we denote them as IEQ, for short. Appearance of IEQs in previously regions is determined by the temporal rate of the stress change, by a proximity of the existing stresses to critical ones in the region, i.e. by concrete geological conditions. Nikolaev and Galkin [28] edited a special collection of papers on IEQs within the former Soviet Union and elsewhere.

Best studied is the induced seismisity observed at the gas mining area Lac in South-Western France near Pyrenees. The mining was started in 1957. The first quake with the magnitude $m = 4.1$ has been registered only in 1969. A very detailed information was given by Volant and Grasso [29], hereafter VG. The area of the gas reservoir is 200 km^2 and it is found at depths from 4 to 6 km. The quakes were observed in layers above, as well as below the reservoir. From 1974 to 1993 about 1000 quakes were registered. VG studied the fractal properties of the IEQ distribution over the reservoir area of 412 IEQs chosen out which 83 cover homogeneously this area while 329 form several clusters in it. Their magnitude-frequency relationships of the G-R kind are:

$$\lg N(\geq m) = a - bm. \tag{22}$$

They estimated $b = 1.026$ for the first group of 83 quakes while for the second larger group a change of the slope at $m = 34$ was found. For $1.7 \leq m \leq 3$ they obtained $b = 0.12$ and $b = 1.15$ for $3 < m \leq 4.2$. We note that for the first group of IEQ the value of b is the same as for the natural EQ where $b \approx 1$, while the second group with the slope change has values of b smaller than the natural EQs. This may be the consequence of the growing power G, see below eq. (23) and discussion of landslide statistics in Sect. 5.7. According to VG the IEQ there have the stress drop $\Delta\sigma \approx 0.2$ MPa = 2 Bar, about 20 times smaller than for natural ones on the average.

The knowledge of the stress drop and magnitude allows us, using eqs. (3, 11, 13), and (2), to estimate the geometrical properties of the faults forming in the process of IEQ such as its length $l \approx 2L$, area $S \approx 0.5L^2$, and the slip displacement at the fault $u = M/\mu S$. These values for the above mentioned magnitudes m are listed in Table. To calculate the slip we used $\mu = 3 \times 10^{10}$ Pa.

TABLE. Characteristic parameters for the faults in IEQ

m	M, N m	l, m	S, m^2	u, cm
1.7	3.55×10^{11}	240	7×10^3	0.2
3.0	3.16×10^{13}	1080	1.2×10^5	0.8
4.2	2×10^{15}	4300	2×10^6	3.3

The smaller value of the stress drop increase here the fault length by a factor $20^{1/3} = 2.7$, the area by a factor 7.4 and by the same factor decrease the slip comparing with natural quakes of the same magnitude. Evidently the stress drop is a function of local geological and tectonic conditions.

As VG correctly note, the changing gas pressure, P, in the reservoir is the engine which drives the seismicity. Since 1985 the rate of P change slowed down considerably and, accordingly, the level of seismicity has dropped. During the first decade since 1974 the pressure P has decreased, according to Fig. 7 from VG by 15 MPa which translates into $\dot{P} = dP/dt = 0.5$ Pa sec^{-1}. This is the rate of change at the roof and bottom of the reservoir. Pressure change corresponds to change of the internal, here elastic, energy per unit volume. The total power causing the stress changes in the crust is, therefore, estimated as hSP, where h is the thickness of the layer under the stress. As a result instead of the total heat power, the geothermal flux in (16), we here have the mechanical power spent on the stress generation:

$$G = hS\dot{P} = V\dot{P}, \tag{23}$$

where $V = hS$ is the volume of material under the stress.

We may use the fact found by VG that the value of b in (22) is close to 1, therefore the form of the G-R law as in (17) would allow us to estimate the value of the numerical coefficients in (17) as we rewrite it here in the form with account of (23).

$$\dot{N}(\geq M) = \frac{lGM^{-2/3}}{h(\Delta\sigma)^{1/3}} = \frac{cS\dot{P}}{(\Delta\sigma)^{1/3}}M^{-2/3}.$$

We know from VG that since 1974 to 1993 about a thousand EQs with $m > 1.7$ were registered and since 1985 the seismicity level dropped. Let us assume for a certainty that 600 of them were registered for the decade starting from 1974. We know the value of $\dot{P}$ for the decade. We know from the Table that $M \geq 3.55 \times 10^{11}$ N m. We know the area $S = 200$ km$^2 = 2 \times 10^8$m^2 and the stress drop $\Delta\sigma = 2 \times 10^5$ Pa. So we can calculate the coefficient $c = 0.6$. Thus we can present the form suitable for engineering estimates through the magnitude m,

$$\Delta N(\geq m) = 10^{-m}c_1 S\Delta P, \tag{24}$$

where the increase of the number of IEQ $\Delta N(\geq m)$ is related to the pressure change for the same period and the area of the seismic region S. Here S is in km^2, ΔP in Bars (10^5 Pa), $c_1 = 10^5 c(\Delta\sigma)^{-1/3} \approx 1.0$ for the Lac reservoir.

The pressure drop dependence of the coefficient c_1 is not very strong, it would be always $O(1)$, e.g. $c_1 = 1$ here and $c_1 = 0.3$ for $\Delta\sigma = 4$ MPa, the mean value found above for natural quakes. So, we may recommend to use eq. (24) to estimate the cumulative number of IEQs for other areas if there are estimates of the pressure drop ΔP and of the activated area S. E.g., could we expect at the Lac area a quake with $m = 5$? From (24) we get $\Delta N(\geq 5) \approx 0.3$. That means we could expect one such EQ if the product $S\Delta P$ would be 3.3 times larger than it was, either $\dot{P}$ should be higher and/or the area S should be correspondingly larger.

We now can try to exploit the presence of the slope drop in the IEQ frequency-size distribution at $m = 3.0$ using the results of Sect. 2. For the natural EQs the slope has to change from -1 to -1.5 when the similarity $\Pi_h = O(1)$. The slopes reported by VG are -0.72 and -1.15 before and after $m = 3$. Here we assume also that $\Pi = 1.36$ corresponds to this value of m. Then one gets from $L = (M/\Delta\sigma)^{1/3}$ and $\Pi = L/h$ that $h \approx 0.4$ km. This agrees well with the thickness of brittle layers reported by VG to be of order half a kilometer in the area. This agreement supplies a justification evidence for using the results developed in Sects. 1 and 2 for the natural quakes also for the induced ones.

The frequency-size distributions of the Gutenberg-Richter type are exhibited also by some other phenomena, or models. One can mention conceptual mechanical models of earthquakes first proposed by Knopoff (see e.g. [7] and a host of other works). Similar behavior is observed for acoustic emission pulses when a sample of rock material is stressed omnidirectionally and a variable pressure is also applied along certain axis of the sample [30].

For more than a couple of decades, astrophysicists are talking on starquakes, SQ, in the crust of neutron stars. It is believed that in the process of SQ soft γ-rays are produced in bursts of millisecond range long. The energy distribution of the burst of one particular pulsar, a neutron star, is a power law with the exponent – 1.66 in the differential form [31], or $n = -0.66$ should be in the cumulative form. At the certain value of energy the frequency-size distribution of the bursts has a change of the slope. Golitsyn [15] used this information, estimates of mechanical properties of the crust, and the theoretical framework presented here in Sects. 1 and 2 estimated the parameters of SQ in the manner just done for the induced quakes. The fault lengths were found to be of order of a kilometer, the slip in centimeters, and the time of crack formation in milliseconds. The known number of events of certain energy during particular observational time allows one to estimate the energy source for the bursts. It is related to the changes of the internal magnetic field strength causing stress in the crust. The life time of a star in such an active regime is estimated to be of order of a few thousand years which agrees with other astrophysical evidence.

Let us summarize the results presented here. In all cases considered up to now the general scheme of the processes is the following. There is a source of energy power, thermal or mechanical, which leads eventually to the stress generation within the material. The stresses are accumulating until a certain limit when they are released by an amount called stress drop $\Delta\sigma$. The value of $\Delta\sigma$ is a very small fraction, $10^{-5} \div 10^{-4}$, of the shear modulus of the material μ. The near constancy, or relatively small variability of the stress drop can be understood within the framework of the SOC, self-organized criticality concept [21]. The size, or intensity, or total energy of an event related to the seismic moment M (and to the magnitude m) are its most important characteristics inferred from observations. From the value of M which has the dimension of energy and the stress drop $\Delta\sigma$, the dimension of energy per unit volume, one forms the geometric scales of the EQ process: length, area, volume. These characteristics presented at Figs. 3 and 4 for the natural quakes do show that such scales are good characteristics of what is observed in reality. The frequency-size distribution is a consequence of the relationship between the energy and its power, eq. (18). It is especially simple for very strong events where the whole crust is destroyed and the fracture propagates one-dimensionally. Weaker events form the fracture plane within the crust which propagates two-dimensionally not reaching the crust boundaries [10].The smaller exponent, 2/3 here in G-R law can be explained by the fact that the stress is applied to the whole crust column but the energy is released in a finite volume within it.

4. The energy cycle of the solid Earth and earthquakes

We already could dwell on it in the Sect. 1, but it would be overloaded with material so we decided to do it in a special section also having in mind an importance of the topic in the overall understanding of how our solid planet is functioning. We shall present here some quantitative estimates at various states of the global cycle.

We know the total geothermal flux F, the prime source of power for all geodynamic processes, to be 4×10^{13} W. It is the source of the mantle convection motions. We can estimate what part of this heat power goes into the kinetic energy generation of convective motions G. This fraction is estimated by an equation derived by Golitsyn [32] as

$$\gamma = \frac{G}{F} = \frac{\alpha g d}{c_p}(1 - \mathrm{Nu}^{-1}), \tag{25}$$

where α is the thermal expansion coefficient equal to 2×10^{-5} K^{-1} for the mantle, g is the gravity acceleration, α is the thickness of the convective layer, about 700 km for the upper mantle, $c_p = 700$ J kg^{-1} the specific heat capacity of the mantle material. All these numbers are from McKenzie et al [33] who modelled convection in the upper mantle and found that the Nusselt number, Nu, the ratio of the actual heat flux through the convective flow to the flux which would be through the layer with no motions, is of order 3. With these numbers our formula produces $\gamma = 0.1$. This means that out of the total 4×10^{12} W goes into the generation of the convective motion energy within the mantle. These motions induce stress on the lithospheric plates. Because of inhomogeneity of the motions stresses on the plates differ in direction and value causing variable stresses within the crust of the plates, especially at the boundaries where they concentrate.

In Sect. 1 we quoted Kagan [22] who gave the global estimate of the seismic moment generation rate $\dot{M}_0 = 6 \times 10^{10}$ N m yr^{-1} which can be translated into 12×10^{13} N m s^{-1}. Using eq. (3) we could say that this is equivalent to one EQ with magnitude $m = 3.4$ happening every second somewhere on the globe. Using eq. (8) relating the energy of the seismic waves E emitted from an EQ, with its seismic moment we can estimate that $\dot{E} = 6.3 \times 10^{-5}\dot{M}$ equals to 8×10^{9} W. Kanamori [34] argues that the value of E represents a very variable fraction of the energy released by quakes depending on their type. Just for certainty in numbers let us assume that the fraction is 8%. Then the total release could be of order 10^{11} W. This may be compared to 4×10^{13} W of the total geothermal flux, or better to our estimate of 4×10^{12} W of energy coming into generation of convective motion kinetic energy in the mantle. From our numbers we can only say that the fraction could be in the range of 0.2 to 2% if we compare it to 4×10^{12} W, or an order of magnitude less if we compare it to the total geothermal flux. Less uncertain estimates could come from numerical models of mantle convection with overlaying plates wherefrom one could estimate the rate of stress increase in the crust and, possibly, its concentrations within it.

Let us summarize the discussion. The energy cycle results in terms of the total geothermal flux F. About 0.1 of it goes into the mantle convective kinetic energy generation rate being eventually dissipating into heat. About 2×10^{-4} of it goes into seismic waves being in itself of order 0.1 or less of the energy spent into all earthquake processes such as stress build-up and its release, rupture, mass dislocations etc. To the author's knowledge these are the first estimates of the solid Earth energy cycle components.

5. What is common among Gutenberg-Richter law and many other natural phenomena?

5.1. General remarks

We start with an obvious relationship, as eq. (18):

$$B(\tau) = \dot{B}\tau, \tag{26}$$

where $\dot{B} = dB/d\tau$. It may have many meanings. From the mathematical point of view one can consider it as the first term in the Taylor expansion series. If B is a linear function of time τ it is exact. It is obvious as a dimensional relationship though in practical applications one may not recall this fact. An example, trivial already since 1687, is that if B is the velocity then $\dot{B} = a$ is the acceleration, and (25) expresses the second law of Newton for the velocity change during time τ:

$$v = a\tau. \tag{27}$$

Many natural phenomena have a statistical nature and reach a steady state as a balance between the forcing and energy dissipation by one way or another. This requires consideration of energy or intensity of the processes in study, i.e. their quadratic characteristics. For this we multiply both sides of eq. (26) by $B(\tau)$ and express the result as

$$B^2(\tau) = \varepsilon\tau, \quad \varepsilon = \frac{d}{dt}\frac{B^2}{2}. \tag{28}$$

The value of ε is the rate of energy, or intensity, supplied into the system, which have to be somehow dissipated, or its forcing. Here we shall distinguish between the forcing, the rate of excitation of quadratic quantities and the force or acceleration per unit mass determining the time behavior of the quantities themselves.

5.2. Acting forces

A steady state can be achieved in various ways. One way, a system has an intrinsic time scale determined by external parameters governing the system. A simple way is to demonstrate it is for the hydrodynamics of viscous fluids governed by the Navier–Stokes equation. In Sect. 1 we presented the Reynolds number in eq. (7) determining two flow regimes: laminar and turbulent, depending on its value.

Multiply and divide r.h.s. of eq. (7) by external length scale r. Then it can be presented as the ratio of two time scales:

$$\mathrm{Re} = \frac{\tau_v}{\tau_d}, \quad \tau_v = \frac{r^2}{\nu}, \quad \tau_d = \frac{v}{r},$$

where τ_v is the viscous time scale and τ_d is the dynamic scale. When $\mathrm{Re} < \mathrm{Re}_{\mathrm{crit}} = O(10^3)$ the flow is laminar and can be described by the linearized Navier-Stokes equation. We can see that in this case $\tau_\nu < \tau_d$ (more precisely $\tau_\nu < \tau_d\,\mathrm{Re}_{\mathrm{crit}}$). Let us consider a pipe of length l and diameter r and pressure difference Δp over the distance l. The force per unit mass, or acceleration, would be then $a = \Delta p/\rho l$. The time scale τ_v is determined by the pipe radius r and kinematic viscosity of the fluid ν. From eq. (27) we now have

$$v = c\frac{\Delta p}{\rho l}\frac{r^2}{\nu}$$

with a numerical coefficient c which may or may not depend on the Reynolds number. In the linear case the problem was solved analytically by Poiseuille in 1837 who found the profile $v = v(r_i)$, r_i the radial distance counted from the pipe axis to its wall where $r_i = r$. For the velocity averaged over the pipe cross-section $c = 1/8$. The mass flux through the circular pipe $Q = \rho S v$ would then be $(\pi/8)\Delta p(l\nu)^{-1}r^4$.

For a pipe with larger diameter r, such that $\mathrm{Re} > \mathrm{Re}_{\mathrm{crit}}$ we use in (27) the dynamic time $r/v = \tau_d < \tau_\nu$. Its value depends on the a priori unknown velocity v. This reflects the non-linear nature of the flow which is now turbulent. Despite of this we can resolve eq. (27) for to v and obtain for the measured mass flux through the pipe

$$Q = c(\rho\Delta p/l)^{1/2}r^{5/2}, \quad v = (r\Delta p/\rho l)^{1/2}.$$

The coefficient $c = 2\pi/\lambda$, where λ is the resistance coefficient, measured in great details and found to be a weakly decreasing function of the Reynolds number (see e.g. [16] This means that our coefficient c is weakly increasing with Re: for $\mathrm{Re} \le 10^5$ the mass flux $Q \propto r^{2.7}$ instead of $r^{2.5}$ if c is constant.

These two examples of the flow regimes in pipes demonstrate well several points. First, an applicability of eqs. (26), or (27), to complicated cases in clarifying their dependence on external, or given, parameters, here the radius and length of the pipe, pressure drop between its ends, nature of the fluids in use. Second, we can not produce the exact value of the numerical coefficient in this way. It requires either the existence of an analytical solution, which can be done in some, especially, linear situations, or calls on numerics but in most important and complicated cases direct experiments. Third, the two examples demonstrate clearly the difference between linear phenomena with externally prescribed time scales and non-linear ones where the scale is itself dependent on the sought values.

Two other examples of linear and non-linear hydrodynamical flows can be found in [15]. One is the derivation of the famous G. Stokes formula for the drag on a sphere moving through viscous fluids. The other relates to the gravity flows both with constant and varying gravity. For the last case the time scale is the pendulum time with possible modification by varying medium density or slope of underlying surface. Of course, the numerical coefficient should be determined separately.

5.3. Active forcings and steady states

Now we turn to forcings and steady states or regimes and consider two cases of fame: the Kolmogorov-Obukhov theory of turbulence and the universal part of the sea surface wave spectrum as developed by many starting with O.M. Phillips [35]. Both theories cover somewhat limited range of spatial and temporal scales and we will show how and in what sense they follow from eq. (26), or, more precisely, from (27) being the consequences of the Newton second law. In both cases all complexities arise from inhomogeneities of phenomena at large scales. In order to avoid this A.N. Kolmogorov [36] proposed to limit oneself by considering only those spatial and temporal scales in which the time and spatial increments of velocity fields can be considered as stationary, homogeneous, and isotropic. For this he proposed to use structure functions, instead of correlation ones. The mathematics of these tools though simple in principle for the fields is quite lengthy and tedious. It can be found in the second volume by Monin and Yaglom [37] (furthermore MY). We limit here ourselves by considering, say, moduli of velocities. Then the structure function is determined by

$$D_v(\vec{r},\tau) = < |v(\vec{x}+\vec{r},t+\tau) - v(\vec{x},t)|^2 > \tag{29}$$

Now we have to determine what is the forcing here. In this case

$$\varepsilon = \frac{d}{d\tau}\frac{v^2}{2} = v\frac{dv}{dt} \equiv v\dot{v} \equiv va$$

i.e. the rate of change of kinetic energy per unit mass, or the product of velocity and acceleration which is the force per unit mass. The last thing is the power as it is taught in the high school physics course. A.N. Kolmogorov, and that was his great point in 1941, assumed that in well developed turbulence flows the values of the energy input into the flow can be considered constant, i.e.

$$\varepsilon = < \frac{d}{dt}\frac{v^2}{2} > = < va > = \mathrm{const},$$

an external parameter coming from instabilities at large scales. For our case we can represent eq. (28) after averaging over an ensemble of Lagrangian particles as

$$< v^2(\tau) > = \varepsilon\tau \tag{30}$$

This expression was known in early 1940s to A.M. Obukhov and L.D. Landau (see [38] and the history in MY). E.A. Novikov [39] showed how it could be obtained from the Langevin type equation with stochastic short-correlated forces. Here it is a direct consequence of (27) and (28) with Kolmogorov's hypothesis that the power input is constant in a statistical sense being equilibrated by viscous dissipation.

Now multiply both sides of eq. (30) by τ^2 and observe that $< v^2(\tau) > \tau^2 = < r^2(\tau) >$, just because $v\tau$ is the distance. We now have

$$< r^2(\tau) > = \varepsilon\tau^3. \tag{31}$$

The dependence $< r^2 > \sim \tau^3$ was first found from observations in the atmosphere by L.F. Richardson [40] and on the ocean surface by Richardson and Stommel [41], Okubo and Ozmidov [42], see also [43]. R.V. Ozmidov [44] collected a large volume of data supporting the dependence (31) from meters to a few thousand kilometers on the ocean surface. Differentiating (31) with respect to time, naming the l.h.s. of it by the turbulent diffusion coefficient $K = 3\varepsilon\tau^2$ and expressing time τ here from (31) as

$$\tau = [< r^2(\tau) > /\varepsilon]^{1/3} = (\bar{r}^2/\varepsilon)^{1/3}, \tag{32}$$

we obtain

$$K = \varepsilon^{1/3}\bar{r}^{4/3}. \tag{33}$$

This expression was first obtained by Obukhov [45] on dimensional grounds, though the $r^{4/3}$ dependence was discovered by Richardson [40]. Substituting τ from (32) into (30) we find that

$$< v^2(\tau(\bar{r})) > = C(\varepsilon\bar{r})^{2/3}, \tag{34}$$

the main result of Kolmogorov [36]. The coefficient C involves in itself the transformation from Lagrangian to Eulerian variables and other statistical procedures and can be found only from observations; see again MY where $C = O(1)$. Also a factor $O(0.1)$ shoud enter (6.13).

Another result of A.N.Kolmogorov [46] can be obtained here by multiplying (30) by v. Then we would have

$$< v^3(r) > \approx C_1\varepsilon r \tag{35}$$

where $C_1 = -4/5$ as was found by him for the three-dimensional turbulence from the Navier-Stokes equation for $\mathrm{Re} \gg 1$, i.e. in the so-called inertial interval where the viscous forces are negligible and the energy flows from larger to smaller scales. For the lack of space and overload with material we do not consider here energy reverse cascades for two-dimensional turbulence [47] for which eqs. (30) to (35) also hold and where $C_1 = +3/2$ [48]. The sign plus here means that for 2D–turbulence the energy flows from smaller to larger scales. 2D–turbulence can be also treated in a similar way [15].

For the structure function of (29) type one can define the energy spectrum (see MY) as

$$D(\tau) = 2\int_0^\infty (1 - \cos(\omega\tau)F(\omega)d\omega.$$

For the Lagrangian velocity structure function (30)

$$F_v(\omega) = \varepsilon\omega^{-2} \tag{36}$$

(for simplicity here and afterwards we omit coefficients). In order to transform it into the wave number space $k = 2\pi/r$ we should know the dependence $\omega = 2\pi/\tau = \omega(k)$. This can be found again from (32) as

$$\omega \sim \varepsilon^{1/3}k^{2/3}, \tag{37}$$

and recalling the relationship between frequency and wave number spectra

$$F(\omega)d\omega = F(k)dk \tag{38}$$

we find that

$$F(k) = F(\omega)\left(\frac{d\omega}{dk}\right) = \varepsilon^{2/3}k^{-5/3}. \tag{39}$$

This is the celebrated result first found by Obukhov [45]. We see again that it is a direct consequence of our eq. (28) and the Kolmogorov hypothesis on the nature and constancy of the energy input ε. Again we suppose, on the dimensional grounds, that the transformation from the Lagrangian to Eulerian variables results in the values of numerical coefficients which, anyway, could be found only experimentally.

The Lagrangian velocity frequency spectrum is expressed by (36). As was shown by Yaglom [49], the corresponding Lagrangian acceleration spectrum would be then

$$F_a(\omega) = \omega^2 F_v(\omega) = \varepsilon = \text{const}$$

and the wave number spectrum as it follows from eqs. (37)–(39) would be

$$F_a(k) = \varepsilon^{2/3}k^{1/3}$$

peaking up at the Kolmogorov microscale

$$l_v = 2\pi/k_\nu = (\nu^3/\varepsilon)^{1/4}.$$

The constancy of $F_a(\omega)$, at the inertial interval of frequencies $2\pi/k_\nu < \omega_\nu = (\varepsilon/\nu)^{1/2}$, where U and L are the external scales for velocity and flow size, means that the Lagrangian accelerations at this interval are of the "white noise" form. This was clearly demonstrated by Novikov [39]. As it is well known, white noise corresponds to the delta-time correlated functions, $\delta(\tau)$. We can say that the assumption (28) with $\varepsilon = \text{const}$ corresponds to the white noise forcing of the flow in Lagrangian description and starting from this point working out backwards we could obtain all the Kolmogorov-Obukhov results.

At the microscale level $r < l_v$, the motions are still stochastic but their velocity structure function can be determined exactly as has been done by Kolmogorov [46]. The structure function, as the mean relative kinetic energy of two particles, separated by the distance r, should be related to an increment of the kinetic energy of a particle traveling the distance r, i.e. $\tau_\nu = r^2/\nu$, the viscous time scale, and then

$$D_v(r) = C_\nu \varepsilon \tau_\nu = C_\nu (\varepsilon/\nu) r^2,$$

where C_ν=1/15 for the velocity components along the position vector r connecting the two points of observation, 2/15 for the components normal to r, and 1/3 for the kinetic energy change.

A striking example of the statistical regularities considered here for turbulence presents us the structure of the velocity field within the body of moving glaciers. The structure was measured by Kazansky [50, 51] at two Pamir glaciers. The velocities were inferred from displacement measurements of a system of sticks fixed at the ice surface. The overall movement of the glaciers was few meters per day with rms relative velocities of the sticks of few centimeters per hour. The velocity structure function for the movements of sticks was found to obey with a good degree of accuracy to the Kolmogorov's "law of 2/3". Kazansky explained the law in this case by the fact that the moving ice mass consists of pieces of various sizes interacting with each other. One, evidently, can assume here accelerations on the pieces to be weakly correlated in time (and space). Then the velocity field averaged over about 8 hours, time of measurements during the work day, reveals the statistical structure similar to the one of the developed turbulence.

Thus one may expect that the same results might hold for the velocity field structure within landslides, snow avalanches and similar phenomena.

At the end of this subsection we briefly stop at the case where the value of A in (28) is the displacement radius of a particle under stochastic forcing. Then

$$< r^2 >= D\tau, \quad D = \frac{d}{d\tau}\frac{r^2}{2}. \tag{40}$$

This is a classical case of random walk diffusion of a Brownian particle. For the diffusion coefficient in the case of spherical particles of radius a A.Einstein found that $D = kT/6\pi\eta a$, where $k = 1.38 \times 10^{-16}$ erg K^{-1}, η is the dynamic viscosity. In this respect we may recall that A.M. Obukhov [52] called the eq. (30) the random walk diffusion in the velocity space, because $< v^2 >= \varepsilon\tau$ with ε playing the role of the diffusion coefficient in this particular space.

5.4. Convection

For the convective flows, the rate of kinetic energy generation is the buoyancy flux b_1, as expressed from eq. (25). The rate of the energy dissipation, ε, is equal to b for steady states, or a part of it for developing flows. The flow in the mantle with the Reynolds number Re $\sim 10^{-20}$ is completely determined by the viscosity, or viscous time τ_ν. The linear scale here is the thickness of the mantle layer, $H \approx 10^3$ km. With the mantle parameters of Sect. 4 and eq. (25) we have $b = 1 \times 10^{-12}$m^{2}s^3. From (40) now we have

$$v_\nu = C'_\nu (b/\nu)^{1/2} H.$$

With $C'_\nu \approx 0.1$ [32] and 1 yr $= 3 \times 10^7$ s, we have now $v_\nu = 3$ cm yr^{-1}. We may recall here that lithospheric plate velocities are within the range of 1 to 12 cm yr^{-1}.

For many natural, and technical, phenomena the rotation is an important factor. The measure of the importance is the Rossby number

$$R_o = v/f_c r$$

with the Coriolis parameter $f_c = 2\Omega \sin\theta$, Ω being the rotation rate, θ the latitude. The Rossby number can be represented as the ratio of the rotation time scale $\tau_f = f_c^{-1}$ and the dynamic time scale τ_d. If $R_o \ll 1$ then $\tau_f \ll \tau_d$ and a flow is in so-called geostrophic balance when the pressure is nearly compensated by the Coriolis force. This is observed for the large scale flows in oceans and atmosphere. It is also the case for the deep convection in polar oceans and in the Earth's liquid core. Then the velocity scale is [53]

$$v = c_f (b/f_c)^{1/2}$$

with coefficient $c_f = 1.7$ [54, 55]. With the same buoyancy flux as for the mantle we obtain for the Earth's liquid core $v \approx 5$ km yr^{-1}. This value is consistent with the observed drift of non-dipole components of geomagnetic field and is enough for geodynamo.

5.5. Sea surface waves

Sea waves represent one of the most fascinating natural phenomena. Just recall the famous print by Hokusai of the Mt. Fudji seen trough the breaking wave! But the mathematical description of them is still elusive. The statistical and similarity kind of description started with the work by Phillips. The results of the long and extensive observational and theoretical work can be briefly described as follows. The measured vertical displacement spectra of the waves in the frequency interval higher than the frequency of the maximum in the spectrum is of the universal form [35, 56, 57]:

$$F_d(\omega) = \alpha g u_* \omega^{-4} \tag{41}$$

where α is a coefficient, $O(0.1)$, g the gravity acceleration, u_* the drag, or dynamical, velocity of the wind. We immediately see that the product of the acceleration and velocity scale is the power input from the atmosphere into the water. We have all the grounds to use the notation $\alpha g u_* \equiv \varepsilon$. The ω^{-4} dependence was first obtained by Zakharov and Filonenko [58] who applied a Hamiltonian formalism for description of the waves and showed how many-wave (4–wave) interactions redistribute the energy over the frequency spectrum.

From (41) we immediately obtain the vertical velocity and acceleration spectra as

$$F_v(\omega) = \omega^2 F_d(\omega) = \varepsilon\omega^{-2}$$

$$F_a(\omega) = \omega^2 F_v(\omega) = \varepsilon = \mathrm{const},$$

just the same as for the Lagrangian turbulence spectra. Only the dispersion relationship here is exact and different from (37) being $\omega^2 = kg$, resulting in the spatial spectra different from the turbulent ones. Because the incompressibility of water, i.e. div $v = 0$, the spectrum of the horizontal velocity component is also $\varepsilon\omega^{-2}$ and the velocity structure function is proportional to τ. This is our starting eq. (28) for the case of constant stochastic forcing. Therefore it should be $< r^2 > \sim \varepsilon\tau^3$ as in (31) with the turbulent diffusion coefficient $K \sim \varepsilon^{1/3} r^{4/3}$.

For the scales larger than about 100 m, the wavelength of the sea waves with maximal energy, we could hypothesize that it is the scale of the maximal energy input into water from the wind and from there up in space scales the turbulence is of a two-dimensional nature transporting the energy upscale. This, as in the atmosphere, would also lead to the $K \sim \varepsilon^{1/3} r^{4/3}$ dependence of the diffusion coefficient. We see once more that our eq. (28) is again for the temporal statistics and its effects for the sea surface waves. The detailed discussion of the Richardson law both in the atmosphere and ocean can be found in [59].

5.6. Statistics of events and G-R law

Starting to look into this problem we can use again the equation (28). Let us represent it for convenience in the redesignated form

$$M = c_m \dot{M} \tau(\geq M) \tag{42}$$

with numerical coefficient c_m. We usually know the size or intensity of an event M from measurements. We may have ideas or know exactly the forcing $\dot{M}$. And we are interested in the cumulative expectation, time $\tau(\geq M)$. The cumulative frequency of events would be from (42)

$$N(\geq M) = [\tau(\geq M)]^{-1} = \frac{\dot{M}}{c_m M}.$$

The inverse magnitude behavior, M^{-1}, is known for many phenomena. E.g., V.F. Pisarenko and M.V. Rodkin gave to the author a graph of cumulative number of tsunamis registered at Sakhalin depending on their height, h. The value gh is a measure of the potential and kinetic energy of the tsunami and all other sea waves. They express the dependence as $\lg N(\geq h) = -0.44 \ln h+$ a number. Recalling the relationship between decimal and natural logarithms as $0.4343 \ln a = \lg a$ we see that the "b" value here is $-0.44 : 0.4343 = -1.01$, i.e.

$$N(\geq h) \propto h^{-1.01} \approx h^{-1}.$$

As we saw in Sects. 1 and 2 the very strong EQs or EQs in thin plates also have the dependence $N(\geq M) \propto M'$ with n close to unity from observational statistics. Because height of tsunamis should be directly related to the EQ size, i.e. to its seismic moment, then the $N(\geq h) \sim h^{-1}$ dependence should mirror the frequency-size distribution of their generators: EQs in thinner sea floor plates when $b = 3/2n \approx 1.5$ as was found by Okal and Romanowicz [3].

We have argued in Sects. 1 and 2 how and why G-R law has two slopes depending on the value of the similarity parameter $\Pi = (M/\Delta\sigma)^{1/3} h^{-1}$, the ratio of the length scale in EQs to the plate thickness h: for $\Pi_M < 1$

$$N(\geq M) \propto M^{-2/3}, \quad \text{or} \quad N(\geq S) \propto S^{-1},$$

where $S \sim (M/\Delta\sigma)^{2/3}$ is the fault area and for $\Pi_M > 1$

$$N(\geq M) \propto M^{-1}.$$

Both cumulative distributions are different forms of our basic eq. (42).

5.7. Omori law and landslides statistics

These two are the examples of distributions for events whose forcing varies with time, and explanation we propose here. The law of Omori (see [23]) says that the frequency of aftershocks (or waiting time for them) decreases (increases) with time counted from some time after the main shock as t^{-p} (or t^p) were p is slightly more than 1. The reverse law is for foreshocks. As concerns to the landslides, two recent papers [60, 61] indicate that cumulative frequency-size distributions have exponents $n = -1.16$ for New Zealand and $n = -1.10$ for Alps. These authors do not present ranges for their exponents but the analyzed number of events is not too large as one could see from their graphs and one could add $\pm$ few hundreds to both numbers. What is important, that anyway $n > 1$ for both areas.

In the process of aftershock formation we have a decrease of their intensity as $M_i = M_0(t_0/t)^\beta$, $t \geq t_0$, $\beta > 0$ and also may have a regional decrease of the stress generation. We approximate the latter as $G_i = G_0(t_1/t)^\alpha$, $t > t_1$, $\alpha > 0$. We may introduce interevent time as

$$\tau_r = \frac{M_0}{G_0} f\left(\frac{G_i t}{M_i}\right) = \frac{M_0}{G_0} f\left(\frac{t}{\tau_i}\right),$$

where index i counts the aftershock number. For larger t/τ_i the function $f(t/\tau_i)$ should tend to a constant, the time between main events. For small t/τ_i, we can expand the function into the Taylor series and take only the first expansion term. Then $\tau_r \approx (t_0/t)^{\alpha-\beta} \propto t^{\beta-\alpha}$. If $\beta > \alpha$, i.e. the aftershock moment decreases faster than the generation rate does, we would have and increase of the aftershock waiting time for an event of the same size. The value of the exponent α could be zero, i.e.

the stress generation rate would not change, then $\tau_r \propto t^{\beta}$. After the main shock and its stress drop the total amount of elastic energy in the area has decreased and its redistribution in space can cause the shocks of only decreasing energy.

For landslides, the cumulative frequency-area size distribution

$$N(\geq A) \propto A^{-n}, \quad n = 1 + \alpha,\ \alpha > 0$$

may be interpreted as a decline in the generation rate of the landslide material, e.g. due its faster spending in the events than its renewal due to weathering processes. Let us approximate eq. (28) in the form

$$A = \dot{A}_0 \left(\frac{t_0}{t}\right)^{\alpha} t(\geq A), \quad t \geq t_0, \quad \alpha \geq 0.$$

This implies the frequency-size distribution in the form

$$N(\geq A) = \left[\frac{\dot{A}_0 t_0^{\alpha}}{A}\right]^n, \quad n = \frac{1}{1-\alpha}.$$

If, e.g., $\alpha = 0.1$ then $n = 1.11$, and for the waiting time we would have

$$\tau(\geq A) = \left[\frac{A}{\dot{A}_0 t_0^{\alpha}}\right]^n$$

and if $\alpha > 0$ then $n > 1$. This means that with a decreasing forcing the mean waiting time increases for an event of the same size. Evidently, after some long enough period the rates of weathering and spending new material should equilibrate, which means $\alpha(t) \to 0$ and then again $\alpha \to 0$ but at a lower level of $\dot{A}$. One may expect some kind of cyclicity in the process of α changes. Eventually after some geological time the process of landsliding in the area under consideration should cease, as we may observe in areas of very old mountains. If forcing is increasing with time then $\alpha < 0$ and $n < 1$ as it may be the case for induced earthquakes.

5.8. Cosmic rays spectra

The cosmic rays, CR, cumulative spectrum, $I(\geq E)$, is also of the form (1) with $b \approx -1.7$ for the interval of particle energies $E = 10^{10} \div 3 \times 10^{15}$ eV and $b \approx -2.1$ for $3 \times 10^{15} \div 10^{18}$ eV [62]. The spectrum is the mean number of particles with energies $\geq$ given energy E measured in unit time per unit area from a unit spherical angle (steradian). CR in the two above ranges are mainly of our Galaxy origin with supernova explosions as their generators with the rate G. It is believed that CR are accelerated stochastically in the mechanism proposed by Fermi. This reminds us of our eq. (28) also relevant here and represented as

$$E \sim G\,\tau(\geq E) \tag{43}$$

In the attempt to explain the first part of the spectrum by similarity and dimensional arguments Golitsyn [63] used (43) as a definition of the time scale: $\tau(\geq E) = EG^{-1}$. It simply means, as in all other phenomena here, the larger the energy, the longer the mean waiting time, or the time to reach the energy. Because the spectrum is measured at a fixed area, we need to estimate a mean distance among CR particles. For this we have to know the particle energy distribution function $n(E)$. The last is related to the spectrum as

$$I(\geq E) = \frac{c}{4\pi} n(\geq E), \quad n(\geq E) = \int_E^{\infty} n(E)dE. \tag{44}$$

Fortunately we may obtain an estimate of the last quantity from above as

$$n(\geq E) = \int_E^\infty n(E)dE = \int_E^\infty \frac{En(E)}{E}dE < \frac{1}{E}\int_E^\infty En(E)dE = \frac{w(\geq E)}{E} < \frac{w}{E} \tag{45}$$

where $w(\geq E)$ is the volume density of the CR particles and is known to be of order 0.5 eV cm^{-3} [62] for energies in the range of 1GeV to about 10^3 GeV. The value $n(\geq \varepsilon)$ in (45) is the volume concentration. The existence of the volume energy in the process allows one to introduce an estimate of the length scale $l = (E/w)^{1/3}$ as for the earthquakes. Here is another analogy with the G-R law. So the unit area per a particle is

$$S(\geq E) \approx [n(\geq E)]^{-2/3} > \left(\frac{E}{w}\right)^{2/3} \tag{46}$$

which is the area of the cross-section of a cylinder related to a CR particle trajectory, its radius being $[n(\geq E)]^{-1/3}$. Therefore we have a time unit and an area unit which together produce the spectrum

$$I(\geq E) \leq \frac{G}{E}\left(\frac{w}{E}\right)^{2/3} = Gw^{2/3}E^{-5/3}. \tag{47}$$

Because of the sign $>$ in (46) this is an estimate from above. However the exponent $5/3 = 1.67$ is very close to the empirical one 1.7. The so-called "knee" at 3×10^{15} eV is explained by the fact that the Larmor radius of a particle with this energy in a magnetic field (whose energy density is of the same order as for the CR particles) starts to feel the finite thickness of the Galactic disc. The constancy of the volume energy of such particles $w(\geq E)$ is now too crude an approximation. But we can estimate, again from above, this value using our eq. (47) and (44)–(46) as for the larger interval of energies, i.e. $E > 3 \times 10^{15}$ eV:

$$\frac{w(\geq E)}{E} > n(\geq E) = \frac{4\pi}{c}I(\geq E) \sim \frac{4\pi}{c}Gw^{2/3}E^{-5/3}$$

wherefrom the unit area now would be

$$S^{-1}(\geq E) > \left(\frac{4\pi}{c}Gw^{2/3}E^{-5/3}\right)^{2/3} = \left(\frac{4\pi}{c}G\right)^{2/3} w^{4/9}E^{-10/9}.$$

The time unit will be taken again from (43) and then

$$I(\geq E) \sim \left(\frac{4\pi}{c}\right)^{2/3} G^{5/3}w^{4/9}E^{-n}, \quad n = -19/9 = -(2 + 1/9).$$

The exponent found here is very close to the empirical one. We stress again that a success of this exercise, explaining results waiting for understanding about half a century, is in use of the stochastic acceleration mechanism in the form (43) also working in many other cases. The stochastic forcing is supposed to be delta-time correlated which is fully in the spirit of the Fermi mechanism.

5.9. Final summary and discussion of results

In the nature, we know often the rate of generation of something; usually it is the power input into the system in study or accelerations, i.e. forces per unit mass. Then eq. (26) or (28) may be considered as a first approximation. If this something is velocity then (26) is the Newton's second law. In the quadratic form (28) in this case we have the relationship between energy and power. There are two main classes of the processes under consideration. First, we know the r.h.s. of eqs. (26) or (28) and want to estimate that something. Second, we measure the something, i.e. the size of an event, have ideas on its generation rate and want to know the number of events as a function of their size (energy, intensity). In other terms, we want to know mean waiting times of the events as a function of their size. In other words, we want to know the frequency-size distributions, or energy spectra. These are tsunamies, EQs, CR discussed here, and many other can be found.

The first class of processes can be subdivided into two categories. In the first one we can determine time scales inherent in the system. That was here the Poiseuille flow in pipes, winds in planetary atmospheres considered by Golitsyn [12, 14, 64, 65], convection in the mantle and Earth's liquid core, etc. In the second category of the processes the system may have a continuum of time scales but there is an equilibrium between the energy input into the system and the energy dissipation rate. The simplest examples here are the Kolmogorov turbulence and the sea surface waves. So, eq. (26) is the single base of the same (single) architectural design: S.A.D. of the Mother nature for a wide variety of the processes. Therefore within the S.A.D. the Gutenberg-Richter law finds its comfortable place among many other ones. The sad fact, which has a stamp of our too specialized scientific life with a great noise level of irrelevant (to the science) perturbations, is that all written here, especially in Sect. 5, could be said decades ago; all the tools and most of facts were known. The rule of the fastest response first mentioned by myself two decades ago [27] and not elaborated much at the time describe, strictly speaking, only the first class of the processes discussed above where the time is known, or observed. In the frequency-size distributions it, or its inverse value, is a sought parameter. In the statistics of events we tacitly assume that each event is independent on the others which means that their statistics is Poissonian. This assumption has been verified for earthquakes by Gardner and Knopoff [66].

The picture presented here might be considered as the very first preliminary approximation of the complex reality. However we hope that it could be useful, at least, as a methodical tool to better understand phenomena known for long and find a preliminary way to describe new phenomena which the nature is not ceasing to reveal for us.

I express my gratitude to the editors of this volume for asking me to submit something to it. I also thank Prof. V.I. Keilis–Borok, a friend of over 40 years, for his support to my attempts to find a place for seismology in the intentionally simple, in many cases oversimplified, picture of the nature, to show why and in what sense EQs are akin to turbulence, sea waves, cosmic rays etc. The invitation to this volume served as an impetus for me to summarize, re-check and put down the material presented here, otherwise it could wait for years to be completed. In writing this, I had for a time a hospitality of Prof. S.S. Zilitinkevich, Department of Earth Sciences, Meteorology, Uppsala University. The list of people with whom I have discussed the material presented here would be too long because of the wide variety of subjects. Nevertheless I would like to mention with great gratitude two my guides through the field of seismology, first seemed to be incomprehensible to me. These are Drs. Y.Y. Kagan, UCLA, and V.F. Pisarenko, IIEP.

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