III. СЕЙСМИЧНОСТЬ, ПРОГНОЗ И МОДЕЛИ

УДК 550.348

A NEW APPROACH TO CHARACTERIZE DEVIATIONS IN THE SEISMIC ENERGY DISTRIBUTION FROM THE GUTENBERG–RICHTER LAW

V. PISARENKO[1], D. SORNETTE[2,3], M. RODKIN[4]

[1]International Institute of Earthquake Prediction Theory and Mathematical Geophysics, Russian Academy of Sciences, Moscow
[2]Institute of Geophysics and Planetary Physics and Department of Earth and Space Science University of California, Los Angeles, USA,
[3]Laboratoire de Physique de la Matière Condensée CNRS UMR6622 and Université des Sciences, France,
[4]Geophysical Center Russian Academy of Sciences, Moscow

A new non-parametric statistic is introduced for the characterization of deviations of the distribution of seismic energies from the Gutenberg-Richter law. Based on the two first statistical log-moments, it evaluates quantitatively the deviations of the distribution of scalar seismic moments from a power-like (Pareto) law. This statistic is close to zero for the Pareto law *with arbitrary* power index, and deviates from zero for any non-Pareto distribution. A version of this statistic for discrete distributions of quantified magnitudes is also given. A methodology based on this statistics consisting in scanning the lower threshold for earthquake energies provides an explicit visualization of deviations from the Pareto law, surpassing in sensitivity the standard Hill estimator or other known techniques. This new statistical technique has been applied to shallow earthquakes ($h \geq 70$ km) both in subduction zones and in mid-ocean ridge zones (using the Harvard catalog of seismic moments, 1977–2000), and to several regional catalogs of magnitudes (California, Japan, Italy, Greece). We discover evidence for log-periodicity and thus for a discrete hierarchy of scales for low-angle dipping, low-strain subduction zones with a preferred scaling ratio $\gamma = 7 \pm 1$ for seismic moments consistent with previous reports. We propose a possible mechanism in terms of cascades of fault competitions.

НОВЫЙ ПОДХОД К ВЫДЕЛЕНИЮ ОТКЛОНЕНИЙ ГРАФИКА ПОВТОРЯЕМОСТИ ЗЕМЛЕТРЯСЕНИЙ ОТ ЗАКОНА ГУТЕНБЕРГА–РИХТЕРА

В.Ф. ПИСАРЕНКО[1], Д. СОРНЕТТ[2,3], М.В. РОДКИН[4]

[1]Международный институт теории прогноза землетрясений
и математической геофизики Российской академии наук, Москва
[2] Институт геофизики и физики планет и факультет геофизических
и космических наук, университет Лос-Анджелеса, Калифорния, США,
[3] Лаборатория физики твердого тела CNRS и Университета, Ницца, Франция,
[4] Геофизический центр Российской академии наук, Москва

Предлагается новый непараметрический подход к выявлению отклонений эмпирических законов повторяемости землетрясений от закона Гутенберга–Рихтера. Метод основан на использовании двух первых статистических лог-моментов распределения и ориентирован на выявление отличий распределения скалярных величин сейсмических моментов от степенного закона Парето в ограниченном нижним порогом диапазоне событий. Исследуемая в рамках этого метода статистика близка к нулю для степенных распределений Парето с произвольным показателем степени, и отклоняется от нуля для распределений, отличных от степенного закона. Для случая дискретных эмпирических распределений значений магнитуд землетрясений используется соответствующий вариант этого подхода. Предлагаемая статистика рассчитывается для разных ограничений на минимальный размер учитываемых землетрясений, что позволяет получить зависимость величины интегрального отклонения от закона Гутенберга-Рихтера в разных диапазонах изменения величин землетрясений. Метод обеспечивает лучшее выявление изменений характера распределения, чем стандартный метод оценки параметра Хилла или другой известный метод. Новый метод оценки использован для исследования эмпирических распределений величин неглубоких (менее 70 км) землетрясений зон субдукции и срединно-океанических хребтов (по данным Гарвардского каталога сейсмических моментов за 1977–2000 гг.) и ряда региональных каталогов землетрясений (Калифорния, Япония, Италия, Греция). В результате использования метода для совокупности зон субдукции с малым углом наклона были выявлены лог-периодические отклонения характера распределения от закона Гутенберга–Рихтера. Выявленная лог-периодичность эмпирического распределения свидетельствует в пользу существования дискретной иерархичности размеров землетрясений с коэффициентом иерархичности (для величин сейсмических моментов) $\gamma = 7 \pm 1$. Обсуждается возможный механизм развития выявленной лог-периодичности.

Introduction

The famous Gutenberg-Richer (G-R) size-frequency law gives the number N of earthquakes of magnitude larger than m_W (in a large given geographic area over a long time interval) [1]. Translating the magnitude $m_W = (2/3)(\log_{10} M_W - 16.1)$ in seismic moment $M_W = \mu U A$ expressed in *dyne-cm* (where μ is an average shear elastic coefficient of the crust, U is the average slip of the earthquake over a surface A of rupture), the G-R law gives the number $N(M_W)$ of earthquake of seismic moment larger than M_W. The striking empirical observation is that $N(M_W)$ can be modeled with a very good approximation by a power law

$$N(M_W) \sim 1/(M_W)^{\beta}. \tag{1a}$$

The Gutenberg-Richter law (1a) is found to hold over a large interval of seismic moments ranging from $10^{20} \div 10^{24} (m_W = 2.6 - 4)$ to about $10^{26.5}$ *dyne-cm* ($m_W = 7$). Many works have investigated possible variations of this law (1a) from one seismic region to another and as a function of magnitude and time. Two main deviations have been reported and discussed repeatedly in the literature:

1) from general energy considerations, the power law (1a) has to cross-over at a "corner" magnitude to a faster decaying law. This would translate into a downward bend in the linear frequency-magnitude log-log plot of (1a). The corner magnitude has been estimated to be approximately 7.5 for subduction zones (SZ) and 6.0 for mid-oceam ridge zones (MORZ) [2, 3] but this is hotly debated (see below);

2) the exponent β is different in SZ and in MORZ [4]. There is in addition a controversy among seismologists about the homogeneity of β-values in different zones of the same tectonic type. Some seismologists believe that β-values are different at least in several zone groups, others find these differences statistically insignificant.

The authors [2, 3, 5–13] proposed, that the large-magnitude branch of the distribution can be modeled also by a power-like law and that the crossover moment or magnitude between these two distributions can be connected with the thickness of the seismogenic zone. Pacheco et al. [8] claimed to have identified a kink in the distribution of shallow transform fault earthquakes in MORZ around magnitude 5.9 to 6.0, which corresponds to a characteristic dimension of about 10 km; a kink for subduction zones is presumed to occur at a moment magnitude near 7.5, which corresponds to a downdip dimension of the order of 60 km. However, Sornette et al. [10] have shown that this claim cannot be defended convincingly because the crossover magnitude between the two regimes is ill-defined. Pisarenko and Sornette [4, 14] suggested new statistical tests to find deviations of earthquake energy distributions from the Gutenberg-Richter law at the extreme range and used the Generalized Pareto Distribution (GPD) to characterize tails of energy distributions in this range. In particular, using a transformation of the ordered sample of seismic moments into a series with a uniform distribution under the assumption of no crossover and applying the bootstrap method, Pisarenko and Sornette [14] estimated a crossover magnitude $m_W = 8.1 \pm 0.3$ for the 14 subduction zones of the Circum Pacific Seismic Belt. Such a large value of the crossover magnitude makes it difficult to associate it directly with a seismogenic thickness as proposed by many different authors in the past. The present paper can be seen as a continuation of the study begun in [4, 14], but we stress that this continuation is based on a quite different non-parametric approach, in contrast to the parametric methods previously developed.

There is a second important novelty here. Complementary to these previous works emphasizing deviations from the G-R distribution only in the tail, the present paper explores the possible existence of deviations from (1a) elsewhere, that is, in the bulk of the distribution. This becomes possible by our introduction of a new statistic, which turns out to be very sensitive to deviations from a pure power law. Namely, we suggest a non-parametric statistic that is close to zero for "pure" G-R laws, and deviates from zero at energy sub-ranges with large enough deviations from the G-R law. This is a first attempt to address the problem of characterizing deviations in the whole range of seismic moment sizes, including moderate and small events, with the hope of connecting such hypothetical deviations with tectonic and geological particularities of the zones in question.

Deviations of a pure G-R power law can take a priori many shapes. It has been recognized with the development of the concept of fractals [15] that power laws are the hallmark of the symmetry of "continuous scale invariance" (CSI) ([16] and references therein). Deviations of a pure G-R power law thus express some degree of breaking of this CSI symmetry. As for any other symmetry, there are many ways to break the CSI symmetry. One of them is particularly interesting because it constitutes a minimalist way of breaking the CSI symmetry: it corresponds to keeping the scale invariance but only for specific scales organized according to a discrete hierarchy with some fixed preferred scaling ratio γ. The lower symmetry thus obtained is called "discrete scale invariance" (DSI) ([17] and references therein). Going from CSI to DSI corresponds to a partial breaking of the CSI, conceptually similar to the partial breaking of continuous translational invariance in liquids into discrete translational invariance in solids. In the present paper, our new statistic unearths a DSI structure decorating the G-R power law, which is the most apparent for low-strain low-angle dipping subduction zones. This is particularly interesting because it complements from a novel angle with a different data set previous reports of DSI in crack growth [18, 19], rupture and fragmentation [20–24]; and seismicity [23–27]. We elaborate on the implication of this finding in the discussion section.

The organization of this paper is as follows. In section 1, we describe the new statistic tailored for studying deviations from the G-R law and summarize its main properties (for continuous variables such as seismic moments). In section 2, we present a similar technique for catalogs with quantized magnitudes, as they are usually given in seismic catalogs (for instance in 0.1 magnitude units). In section 3, we apply these statistics both to some simulation examples and to the Harvard catalog of seismic moments. In section 4, we apply the discrete version of our statistic to several regional catalogs. Section 5 presents a discussion of our results and concludes.

1. *TP*-statistic and its properties

It is well-known that, in terms of (scalar) seismic moments, the G-R law coincides with the Pareto distribution $F(x)$, allowing us to rewrite (1a) as

$$F(x) = 1 - (u/x)^{\beta}, \quad x \geq u, \quad \beta > 0, \tag{1b}$$

where u – lower threshold, and β – power index of the distribution. Let us consider a finite sample $x_1, \ldots, x_n$. It is desirable to construct a statistic $TP = TP(x_1, \ldots, x_n)$ such that, asymptotically for large n, TP would be close to zero and, at the same time, would deviate from zero for samples whose distribution deviates from equation (1b). Let us construct such statistic based on the first two normalized statistical log-moments of the distribution (1). Using the symbol E for the mathematical expectation, we have

$$E \log(X/u) = \int\limits_u^{\infty} \log(x/u) dF(x) = 1/\beta, \tag{2}$$

$$E \log^2(X/u) = \int\limits_u^{\infty} \log^2(x/u) dF(x) = 2/\beta^2. \tag{3}$$

Thus, if we choose

$$TP = \left(1/n \sum_{k=1}^{n} \log(x_k/u)\right)^2 - (0.5/n) \sum_{k=1}^{n} \log^2(x_k/u), \tag{4}$$

then according to the Law of Large Numbers and equations (2)–(3), the statistic TP tends to zero as $n \to \infty$. In order to evaluate the standard deviation $std(TP)$ of the statistic TP, we rewrite (4) in the form:

$$TP = \left(1/n \sum_{k=1}^{n} [\log(x_k/u) - E_1] + E_1\right)^2 -$$
$$-(0.5/n) \sum_{k=1}^{n} [\log^2(x_k/u) - E_2] - 0.5E_2, \tag{5}$$

where E_1, E_2 are the expectations of $\log(x_k/u)$ and $\log^2(x_k/u)$ respectively (for Pareto samples, $E_1 = 1/\beta$ and $E_2 = 2/\beta^2$). Both sums in equation (5) are of the order $n^{-0.5}$:

$$\varepsilon_1 = 1/n \sum_{k=1}^{n} [\log(x_k/u) - E_1] \propto n^{-0.5},$$

$$\varepsilon_2 = 1/n \sum_{k=1}^{n} [\log^2(x_k/u) - 2/\beta^2] \propto n^{-0.5}.$$

Thus, if n is large enough, we can expand TP in equation (5) into Taylor series up to terms of the order $n^{-0.5}$ in the neighborhood of E_1 and E_2 respectively:

$$TP \cong (E_1^2 - 0.5E_2) + 2E_1\varepsilon_1 - 0.5\varepsilon_2. \tag{6}$$

This provides an estimation of $std(TP)$ by the standard deviation of the sum:

$$2E_1\varepsilon_1 - 0.5\varepsilon_2 = (2E_1/n)\sum_{k=1}^{n}[\log(x_k/u) - E_1] - (0.5/n)\sum_{k=1}^{n}[\log^2(x_k/u) - E_2] =$$

$$= (0.5E_2 - 2E_1^2) + (1/n)\sum_{k=1}^{n}[2E_1\log(x_k/u) - 0.5\log^2(x_k/u)]. \tag{7}$$

The standard deviation of the last sum in (7) can be estimated by

$$n^{-0.5}std[2E_1\log(x_k/u) - 0.5\log^2(x_k/u)], \tag{8}$$

and the standard deviation std of the term in bracket in equation (8) is estimated through its sampled value $[2E_1\log(x_k/u) - 0.5\log^2(x_k/u)]$. Equation (8) provides an estimate of $std(TP)$ if we replace E_1 by its sample analog:

$$(1/n)\sum_{k=1}^{n}\log(x_k/u).$$

Fig. 1 shows the TP-statistic as a function of the lower threshold u, applied to a simulated Pareto sample of size $n = 5000$ with power index $\beta = 2/3$ generated with $u = 1$ as defined in equation (1b). Keeping fixed the synthetically generated data, for a given lower threshold u, we select all data

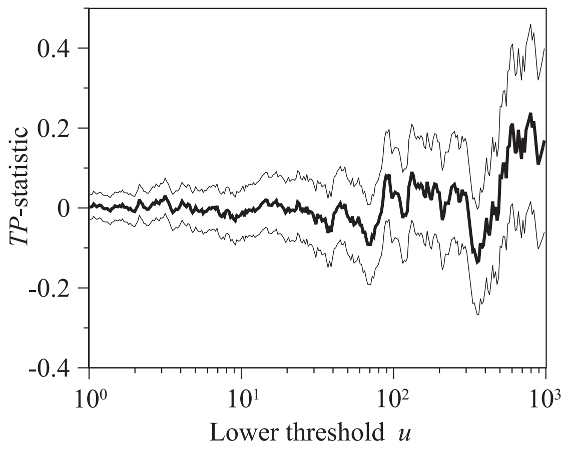


Fig. 1. TP-statistic as a function of the lower threshold u, applied to a simulated Pareto sample of size $n = 5000$ with power index $\beta = 2/3$, generated with $u = 1$ as defined in eq.(1).

Increasing u decreases the number of data values used in the calculation of the statistic TP, thus enhancing the fluctuations around 0. Two thin lines show plus and minus one standard deviation std estimated as exposed in the text values that are larger than u and calculate TP using only these values above u. Varying u allows in principle to test different part of the distribution. If the lower threshold u is smaller than 300, the TP-statistic does not deviate significantly from zero. Beyond this value (for which there are less than 150 data values), random fluctuations become large as TP is estimated with a smaller and smaller number of data.

Having in mind the problem of DSI that we shall encounter in our investigations below, we illustrate the application of the TP-statistic to simulated samples generated by a Pareto-like distribution with log-periodic oscillating deviations from an exact Pareto law. Namely, the chosen distribution function is defined by:

$$F(x) = \begin{cases} 1 - C_1(b, \Delta b)/x^{b+\Delta b}, & k \cdot \Delta l \le \log_{10}(x) < (k + 1/2) \cdot \Delta l, \\ 1 - C_2(b, \Delta b)/x^{b-\Delta b}, & (k + 1/2) \cdot \Delta l \le \log_{10}(x) < (k + 1) \cdot \Delta l, \end{cases} \qquad (9)$$

where $k = 0, 1, \ldots$. The theoretical tail of this distribution function with $b = 0.67$; $\Delta b = 0.2$; $\Delta l = 0.75$ is shown in fig. 2a (Δl is the $\log_{10}$-period of the DSI oscillations, as it is seen from equation (9), corresponding to a preferred scaling ratio $\gamma = 10^{\Delta l} = 5.62$); C_1, and C_2 are normalizing constants depending on b, Δb. With $\Delta b = 0.2$, the oscillations are hardly observable on the graph. Fig. 2*a* shows as well a sample analog of the tail function constructed from a sample of size $n = 5000$ generated with the DF (9).

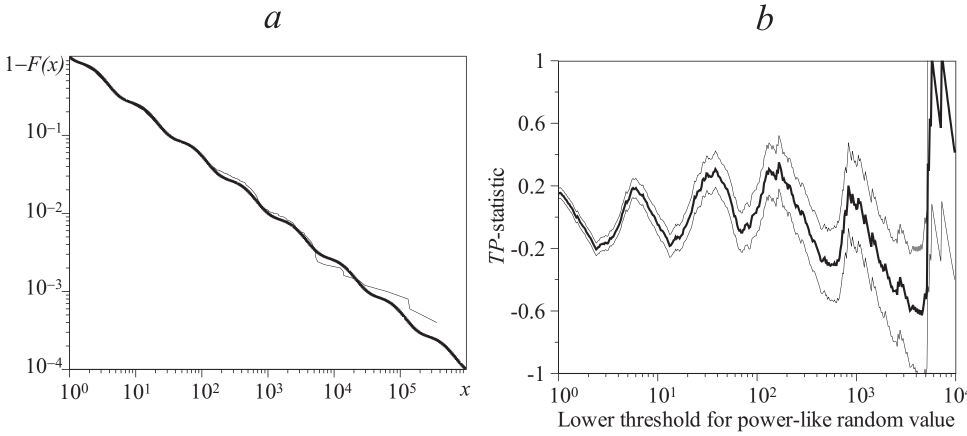


Fig. 2. Model case of the distribution function (9) with regular log-periodic oscillating deviations around a pure power law:

a – theoretical distribution tail (solid line) and sample tail (thin line), sample of size $n = 5000$; *b* – TP-statistic as a function of the lower threshold u, applied to the sample of size $n = 5000$, plus and minus one standard deviation are shown by thin lines.

The parameters are $b = 0.67$, $\Delta b = 0.2$, $\Delta l = 0.75$ (Δl is the $\log_{10}$-period of the DSI oscillations, as it is seen from eq.(9), corresponding to a preferred scaling ratio $10^{\Delta l} = 5.62$)

Fig.2b shows the TP-statistic as a function of the lower threshold u applied to the above sample. We see that the DSI oscillations are very strong and distinguishable despite some noise disturbances (in particular, at large lower thresholds u). Note that the maxima and minima of the log-periodic oscillations shown in fig. 2b correspond to the points of changing slopes shown in fig. 2a: local maxima of fig.2b correspond approximately to the transition from slope $(b + \Delta b)$ to slope $(b - \Delta b)$, and local minima to the transition from $(b - \Delta b)$ to $(b + \Delta b)$.

2. *TED*-statistic for discrete exponential distribution

Many catalogs measure earthquake sizes with discrete magnitudes rather than with continuous seismic moments. Accordingly, the G-R law is an exponential distribution when earthquake sizes are expressed in magnitudes. In this section, we address the problem of constructing a statistic similar to TP for a quantized exponential distribution. Let us consider the (shifted) exponential distribution:

$$F(x) = 1 - \exp(-(x-u)/d), \quad x \geq u, \tag{10}$$

where d is a scale parameter. Let us assume furthermore that the distribution (10) is quantified with a step Δ providing discrete probabilities:

$$\begin{aligned} p_k &= F(u + k\Delta) - F(u + (k-1)\Delta) = \\ &= \exp(-(k-1)\Delta/d) - \exp(-k\Delta/d), \;\; k = 1, 2, \ldots. \end{aligned} \tag{11}$$

Suppose further that, in a given catalog of magnitudes, there are m_k, $m_k \geq 0$, values within the interval $(u + (k-1)\Delta; u + k\Delta)$, $k = 1, 2, \ldots$, so that

$$m_1 + m_2 + \ldots + m_k + \ldots = n, \tag{12}$$

where n is the total number of observed magnitudes (sample size). We can consider the sample $(m_1, m_2, \ldots m_k \ldots)$ as the result of n independent trials of discrete rv ξ taking values $1, 2, \ldots, k, \ldots$ with probabilities (11). The first two moments of the rv ξ are :

$$\begin{aligned} M_1 &= \sum_{k=1}^{\infty} k p_k = 1/(1 - \exp(-\Delta/d)), \\ M_2 &= \sum_{k=1}^{\infty} k^2 p_k = (1 + \exp(-\Delta/d)/(1 - \exp(-\Delta/d))^2. \end{aligned} \tag{13}$$

Denoting $\lambda = \exp(\Delta/d)$ we get:

$$M_1 = \lambda/(\lambda - 1), \quad M_2 = \lambda/(1+\lambda)/(\lambda - 1)^2. \tag{14}$$

We derive from the first equation in (14):

$$\lambda = M_1/(M_1 - 1), \tag{15}$$

and from the second equation in (14):

$$\lambda = (M_1 + M_2)/(M_2 - M_1). \tag{16}$$

Replacing the theoretical moments M_1, M_2 by their sample analogs

$$M_1^* = \sum_{k=1}^{\infty} k m_k/n, \quad M_2^* = \sum_{k=1}^{\infty} k^2 m_k/n, \tag{17}$$

in equations (15), (16) we get two different estimates of λ whose difference converges to zero in probability as $n \to \infty$. This results from the fact that M_1^*, M_2^* are consistent estimates of M_1, M_2 for any value of the unknown scale parameter d. This provides the looked-for TED-statistic:

$$TED = (M_1^* + M_2^*)/(M_2^* - M_1^*) - M_1^*/(M_1^* - 1). \tag{18}$$

Our remaining task is to derive a consistent sample estimate of the standard deviation std(TED) of the statistic TED defined by (18). For this purpose, we use the formulae derived in [28] for the variance of limit normal distributions for sample moments of a multinomial distribution. The variance of the limit normal distribution of TED can be estimated as follows. Let us denote by U_1, U_2 the following statistics:

$$U_1 = 1/(M_1^* - 1)^2 + 2/(M_2^* - M_1^*) + 2M_1^*/(M_2^* - M_1^*)^2, \tag{19}$$

$$U_2 = 2M_1^*/(M_2^* - M_1^*)^2. \tag{20}$$

Then, the variance of the limit normal distribution of TED can be consistently estimated by the following expression (see for details [28], chapter 6, section 6a.1):

$$var(TED) \cong \sum_{k=1}^{\infty} k^2 (m_k/n)(U_1 - kU_2)^2 - \left[\sum_{k=1}^{\infty} k(m_k/n)(U_1 - kU_2)\right]^2. \tag{21}$$

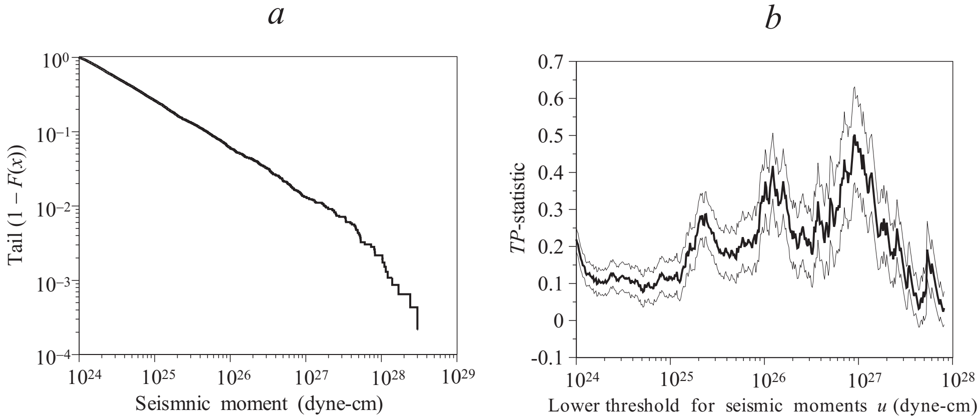
3. Application of *TP*-statistic to the Harvard catalog of seismic moments

Let us consider first the Harvard catalog of (scalar) seismic moments for the time period 1977–2000, and for shallow events ($h \le 70$ km). Only earthquakes with seismic moments larger than 10^{24} *dyne-cm* are considered to ensure a tolerable completeness and homogeneity. The total number of earthquakes in all 14 considered subduction zones (SZ) is 4609. A detailed analysis of the seismic parameters of individual zones is beyond the scope of our paper. We use this sample mainly for illustration of our approach based on TP-statistics with a movable lower threshold. More detailed information on the seismic moment data and seismic parameters of individual zones can be found in [4, 14].

3.1. *TP*-statistic of earthquakes in subduction zones (SZ)

The sample tail of SZ events is shown on fig. 3*a*. Except for the extreme range, the sample tail function $1 - F(x)$ looks like a straight line in double log-scale, i.e. the G-R law seems to apply.

At the very extreme end of the range of seismic moments, a "bend down" can be observed, but there is no strikingly visible deviations from a straight line in the middle part (the careful reader may however notice the existence of an oscillation of very small amplitude). Fig. 3*b* shows the TP-statistic applied to the subduction sample. Regular oscillations on a noisy background are now obvious. In addition, the TP-statistic is translated upward by 0.1–0.2 which is probably the signature of the bend down in the extreme range. By comparing fig. 3*b* with fig. 1, it is clear that the TP-statistic with a moving lower threshold provides a rather sensitive method for detecting deviations from the G-R law. We can already conclude from this analysis that the distribution of earthquake moments exhibits significant deviations from the pure power law, not only in its tail but also, in a major portion of the scaling region. In a quantitative form, this evidence can be considered as a new claim. The comparison of the oscillations observed in fig. 3*b* with those of the synthetic test in fig. 2*b* suggests that the deviation of the distribution of seismic moments from the G-R law can be modeled by log-periodic oscillations, reflecting a partial breaking of CSI into DSI. For comparison, we show in fig. 3*c* the so-called Hill's estimates of the power index β as a function of the moving lower threshold u (see, e.g. [29], for details on the Hill's estimators). The oscillations of the power index are hardly observable on this graph.



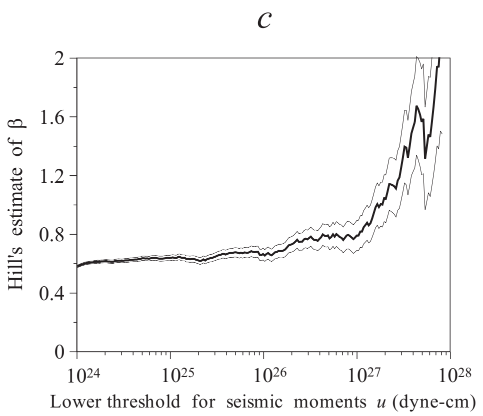


Fig. 3. Log-periodic deviations in the distribution of seismic moments for subduction zones (SZ) earthquakes from the Harvard catalog for the time period 1977–2000 and shallow events ($h \le 70$ km):

a – the tail of empirical distribution function of (scalar) seismic moment vales; b – TP-statistic applied to these data; c – the Hill's estimates of the power index β as function of the moving lower threshold u, applied to the same data. At b and c figures plus and minus one standard deviation are shown by thin lines.

Only earthquakes with seismic moments larger, than 10^{24} (dyne-cm) are considered to ensure completeness and homogeneity; sample of size $n = 4609$

3.2. DSI in the TP-statistic of earthquakes in subduction zones

Let us come back to the oscillations observed in fig. 3b for the distribution of event sizes in subduction zones. While they are significant, can they be related to any geophysical characteristics? We are going to suggest such a tentative geophysical interpretation if these oscillations are genuine, acknowledging in the same time that a definite conclusion on this subject needs more detailed study based on a more representative data.

The seismic and stress-strain regimes in active transitional zones are believed to be determined mainly by the mechanical coupling between the down going slab and the overriding continental plate. The dip angle is known as one of the factors affecting the mechanical coupling [30]. It seems reasonable to suggest that the increase in the mechanical coupling promotes both an increasing seismic activity and a more noticeable log-periodicity.

To perform a quantitative analysis, we use a pre-existing classification of subduction zones constructed by Jarrard [30], based on a set of geological and tectonic characteristics (29 parameters), of which we select the following main 5 parameters:
– intermediate dip up to 100 km of depth (in degrees),
– strain class (in an abstract discrete scale from 1 to 7),
– convergence rate (in cm/year),
– mean slab age at trench (in m.y.),
– maximum moment magnitude M_w.

Jarrard [30] provided the corresponding 5 parameter values for each of his 39 subduction zones. To obtain the 5 parameter values for each of our 14 subduction zones, each one often made of several zones considered by Jarrard, we averaged the corresponding parameter values over all Jarrard's zones constituting each of our zones. The resulting values of the 5 parameters for each of our 14 subduction zones are given in table.

Characteristics of subduction zones (selected from Jarrard, 1986)

Zone	Intermediate dip angle, in degrees	Strain class, (I–VII)	Convergence rate, cm/yr	Slab age, m.y.	Maximum observed magnitude M_w
Alaska	18 (L)	VI (H)	6.3 (L)	49 (L)	9.1 (H)
Japan	21 (L)	VI (H)	9.9 (H)	67 (H)	8.6 (H)
Kuril Isls	28 (L)	V (H)	8.7 (H)	119 (H)	8.8 (H)
Kamchatka	25 (L)	V (H)	8.8 (H)	90 (H)	9.0 (H)
Mariana	26 (L)	IV (L)	7.6 (L)	94 (H)	7.2 (L)
Mexico	60 (H)	VI (H)	7.2 (L)	17 (L)	8.4 (L)
S. America	20 (L)	VII (H)	10.0 (H)	38 (L)	9.5 (H)
Sandwich Isls	67 (H)	I (L)	0.9 (L)	49 (L)	7.0 (L)
New Hebrides	44 (H)	I (L)	8.8 (H)	52 (L)	7.9 (L)
Solomon Isls	42 (H)	IV (L)	12.0 (H)	50 (L)	–
New Guinea	35 (H)	I (L)	4.3 (L)	50 (L)	–
Taiwan	41 (H)	–	4.6 (L)	–	–
Tonga	29 (L)	I (L)	7.5 (L)	117 (H)	8.3 (L)
Sunda	21 (L)	V (H)	8.2 (H)	88 (H)	(L)

Values classified as "low" marked by (L), *classified as "high" marked by* (H)

We then consider each of the 5 parameters one by one. For each parameter, we classify all 14 zones into two groups with approximately equal numbers of elements in accordance with "relatively high" (H), or "relatively low" (L) values of the parameter. This provides us with 5 different partitions of 14 zones marked in table I as (L) and (H). We group all earthquakes of the zones in the "high" (respectively "low") group of a given partition and apply the TP-statistic to this "high" (respectively "low") group separately. Fig. 4*a* shows that the group with "low" dips is characterized by significant log-periodic oscillations of its TP-statistic, whereas no oscillation can be observed in the TP-statistic of the "high" dip group (fig. 4b). Similar differences in the TP statistics of the "low" and "high" strain classes are found. The classification using the three other parameters does not give noticeable differences between the H and L classes. This suggests that the log-periodic oscillations are associated with low dip and low strain subduction zones in the middle part of the slab (60–100 km depth). From the measure of the period in the logarithm of the lower threshold, we infer a preferred scaling ratio $\gamma = 7 \pm 1$.

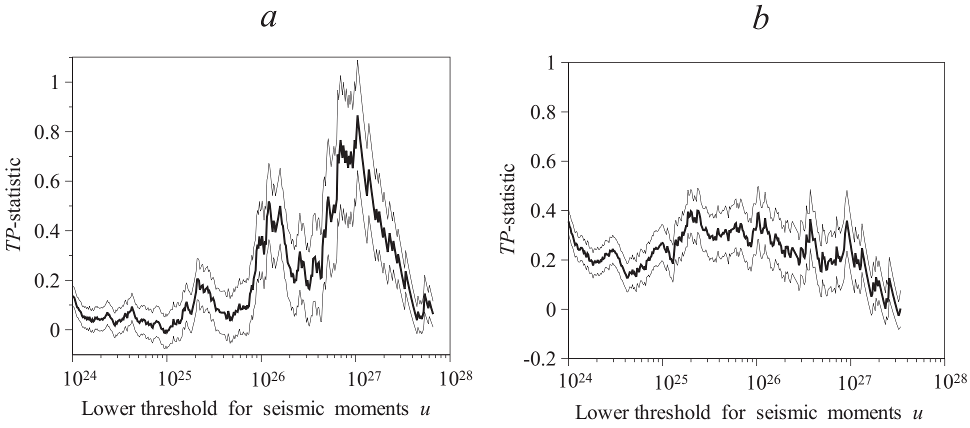


Fig. 4. TP-statistic as a function of the moving lower threshold u for the group of subduction zones with "low" (a) and with "high" dip angles (b).
Plus and minus one standard deviation are shown by thin lines

The origin for log-periodicity may be due to a mechanism similar to that found in growing antiplane shear faults [19] according to the following mechanism. With simple subduction zone plate bending, intraplate outer rise earthquakes are mostly due to tensional failure at shallow depths. The location of the plate bending corresponds to the location of stress concentration constituting the most favorable loci for earthquake nucleation and fault growth. We visualize a system of faults more or less parallel to the subduction boundary. As the plate undergoes its subduction, faults compete with each other to accommodate the growing strain: nearby faults screen each other. This competition between two neighboring faults imply that one of them will start to grow faster and be more active while the other one slows down, being screened by the first one. As this process can occur at all scales, this leads to a cascade of Mullins-Sekerka instability as demonstrated in [19] by analytical as well as numerical calculations: from an initial homogenous population of faults, the cascade of growth instabilities creates a discrete hierarchy of fault lengths with a scaling factor between successive levels of the hierarchy close to 2. Thus, we can assume that earthquakes reveal these discrete hierarchy of faults. As seismic moments are approximately proportional to the cube of the rupture length, this predicts a preferred scaling factor of $2^3 = 8$ for seismic moments. This value is compatible with our measurement $\gamma = 7 \pm 1$. We stress that the mechanism of competition between growing and reactivated faults operates both for normal as well as for transform faults and is not restricted to antiplane shear faults. Our proposed mechanism rationalizes our finding that a DSI fault network should be more apparent for low-strain low-angle dipping subduction: only then can a large delocalized lateral zone of faulting be created with many sub-parallel faults interacting and competing. Large dipping angles localize the region of competing faults to a narrow scale range, preventing the observation of log-periodicity.

We recall that all these considerations are tentative since the number of different subduction zones under analysis (14) is too small to draw a definite conclusion.

4. Application of the *TED*-statistic to catalogs with discrete magnitudes

Let us now illustrate the application of the TED-statistic introduced in section 3 to the global NEIC catalog as well as to the regional catalogs of Japan, California and Italy, all reported with discrete magnitudes (the discrete magnitude bins are in all cases 0.1 of the magnitude unit). The sample magnitude-frequency curves (MFC) are shown on fig.5*a*, and corresponding TED-statistic as a function of the lower threshold u are shown on fig. 5*b,c*.

On the MFC of the global NEIC catalog (fig. 5*a*), there is no visible feature except for a downward bend at the extreme range, starting near $M = 7.5$. On the TED-graph (fig. 5*b*), there is a corresponding increasing TED-statistic with an acceleration at the extreme range. Thus, the tail of the sample MFC deviates more and more strongly downward from the G-R law as one penetrates further in the tail of the distribution. One could say that this global catalog is "too heavily averaged" to find any other significant characteristic deviation from the G-R in the middle part of the range.

The MFC of the California catalog (fig. 5*a*) has a slightly convex form in the range $1 \le M \le 3$ that can be explained by the partial incompleteness of the catalog in this range. This fact is reflected by a positive deviation of the TED-curve in this interval (fig. 5*b*). Above $M \cong 3$, the completeness of the catalog seems satisfactory, and the TED-curve remains near zero till $M \cong 5$, where a smooth increase starts interrupted by two "humps" near $M \cong 5.9; 6.3$ (a "trough" near $M \cong 6.7$ does not seem reliable because of the large *std*). In the central part of the range $3 \le M \le 5$, deviations from the G-R law are small.

Regional catalogs reveal more structures in their TED-graphs. The sample MFC for Japan (fig. 5*a*) is visibly a convex function with a small downward bend at the extreme range. On its corresponding TED-curve (fig. 5*c*), besides the extreme upward bend, one sees two small "troughs" near $M \cong 5.75$ and $M \cong 6.9$. The nature of these troughs is unclear. In the small and intermediate range $3 \leq M \leq 5.75$, the TED-curve is almost horizontal (a small negative shift is probably due to the final downward bend of the MFC). It thus seems that the G-R law for Japan in the middle part of range is fulfilled satisfactorily.

The MFC of Italy (fig. 5*a*) exhibits some deviations caused by the practice of a human operator to prefer magnitude values that are multiples of 0.5. Small local minima corresponding to such magnitude values are hardly

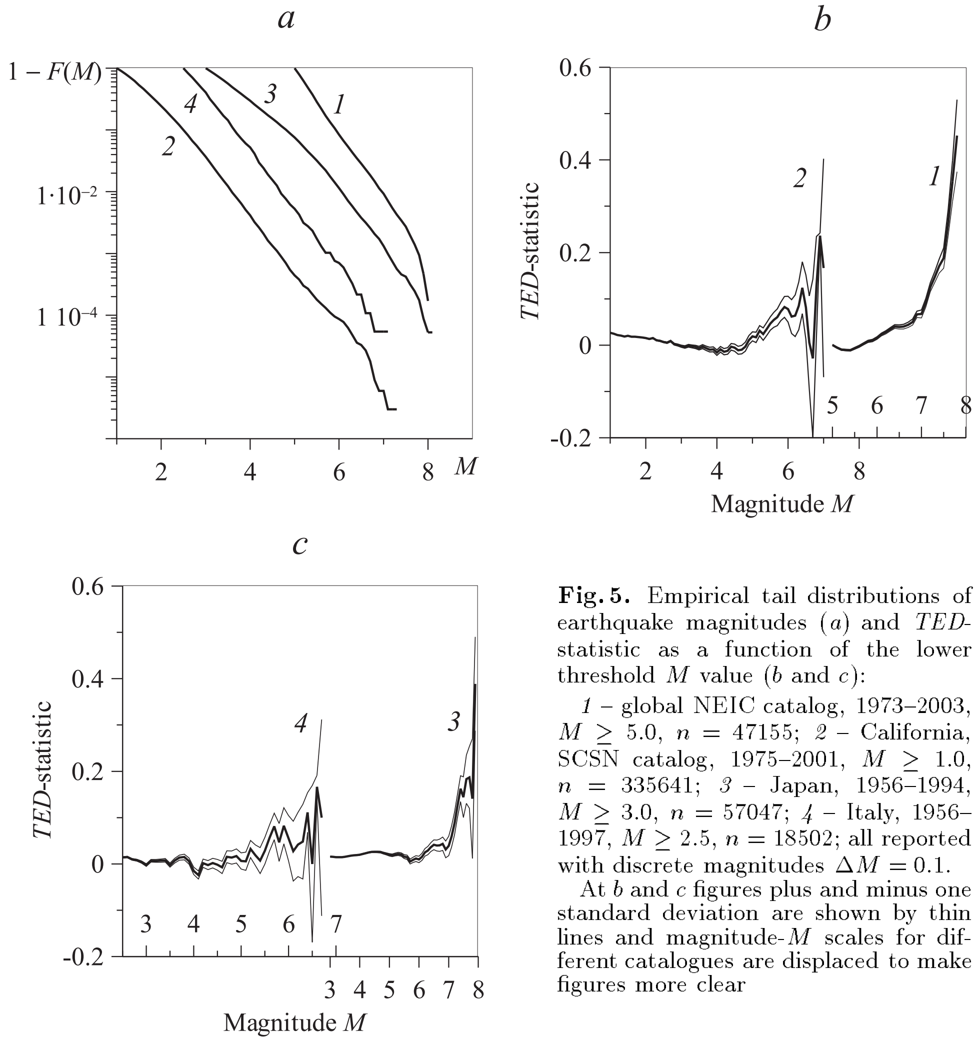


Fig. 5. Empirical tail distributions of earthquake magnitudes (*a*) and TED-statistic as a function of the lower threshold M value (*b* and *c*):

1 – global NEIC catalog, 1973–2003, $M \geq 5.0$, $n = 47155$; *2* – California, SCSN catalog, 1975–2001, $M \geq 1.0$, $n = 335641$; *3* – Japan, 1956–1994, $M \geq 3.0$, $n = 57047$; *4* – Italy, 1956–1997, $M \geq 2.5$, $n = 18502$; all reported with discrete magnitudes $\Delta M = 0.1$.

At *b* and *c* figures plus and minus one standard deviation are shown by thin lines and magnitude-M scales for different catalogues are displaced to make figures more clear detectable in the MFC, but they are clearly observed in the TED-curve (fig. 5c), sometimes with small shifts. Indeed, one can discern small but quite definite minima near $M \cong 3; 3.5; 4.1; 4.6; 5; \ldots$. A "hump" near $M \cong 5.7 - 5.8$ is seen, but it cannot be ascertain due to the large *std*.

Thus, unlike the Harvard catalog of seismic moments, we did not find noticeable oscillations in magnitude catalogs. The four local catalogs (Japan, California and Italy) are characterized by very complex boundaries and we would not expect the simple mechanism of competing faults to be as clearly *apparent* due to the probable effect of averaging over different fault orientation and fault mechanism: indeed, previous studies have shown that log-periodicity can only reveal DSI if adequate steps to prevent the destruction of the oscillations by averaging are taken [23]. The situation is not improved by the use of discrete magnitudes as it is well known that techniques of measuring earthquake size by magnitude is less accurate as compared with the method of seismic moments. Different magnitudes are based on measurements of maximum amplitudes on seismograms registered by seismometers with different frequency ranges. Besides, many magnitude catalogs have had a long evolution of measurement procedures, thus causing some non-stationarity in the registered time-series of observed magnitudes. Quantifying magnitudes with step 0.1 can lead to inaccuracies for the determination of earthquake energies, but probably with not too serious consequences (except for the measurement of log-periodicity). Of course, larger steps, say of 0.25, are able to cause appreciable undesirable effects. To check this, we have converted the Harvard seismic moments (for all 14 subduction zones, $n = 4609$) into discrete magnitudes in units of 0.1 and have applied to them our TED-statistic (see fig. 6). We obtain very weak maxima which, while corresponding to the strong maxima described above for the continuous

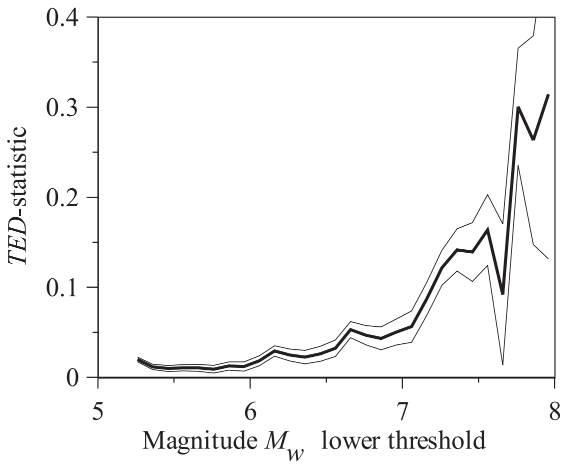


Fig. 6. TED-statistic applied to the Harvard seismic moments (for all 14 subduction zones, $n = 4609$), converted into discrete magnitudes M_W in units of 0.1.
Plus and minus one standard deviation are shown by thin lines

TP-statistic, are barely statistically significant with the TED-statistic. We observe the same phenomenon with synthetic log-periodic power laws of seismic moments, when transformed into discrete magnitudes. Thus, the quantization of magnitudes decreases strongly the efficiency of the detection of log-periodic oscillations. Thus, for detailed statistical analysis of deviations from the G-R law, the Harvard catalog of seismic moments is preferable to the magnitude catalogs, despite the fact that magnitude catalogs cover longer time intervals.

Thus, we can conclude once more that the catalog of seismic moments is preferable for detailed statistical treatments as compared with magnitude catalogs.

5. Discussion and conclusions

We have suggested a T-statistic of a new type measuring quantitatively the deviations from the G-R law expressed both as a function of seismic moments and as a function of discrete magnitudes. This statistic can be displayed as a function of a lower threshold. In this respect, it is similar to the well known *mean excess function* $e(u)$, see, e.g. [29]:

$$e(u) = E(X - u|X > u). \tag{22}$$

This function was introduced as a useful statistical tool to characterize tails of distributions. For power-like tails, it behaves as a straight line with positive slope; for exponential tails, it is a constant, and so on. Our statistic (for continuous distributions) is based on two *mean log-excess moments* $l_1(u)$, $l_2(u)$:

$$\begin{aligned} l_1(u) &= E(\log(X/u)|X > u), \\ l_2(u) &= E(\log^2(X/u)|X > u). \end{aligned} \tag{23}$$

For discrete magnitudes with exponential distribution (the G-R law), in addition to (22), the second *mean excess moment* $e_2(u)$ in discrete form was used:

$$e_2(u) = E((X - u)^2|X > u). \tag{24}$$

These T-statistic based on statistical moments are not local characteristics of corresponding densities, they are characteristics of a *cumulative* type, referring to the tail on the interval (u, ∞). Thus, they do not answer to question "at what location a particular sample differs from the G-R law?", but rather to "what part of the extreme tail differs from the G-R law?".

We gave examples of the application of the T-statistics to several previously well-studied earthquake catalogs. Because of its cumulative property, the T-statistic is, generally speaking, more stable than local characteristics of deviations from a given law. The T-statistic permits to judge on the deviation of an integral tail portion from the G-R law, but it is not tailored specially to work with the limit extreme behavior of the tail. For the latter, we had suggested other methods in [4, 14] and we hope to continue working in this direction elsewhere.

It should be noted that for the Pareto (or, for the exponential) distribution there exist a lot of non-trivial statistics, whose distribution is free of the form parameter β, and that can be used for checking deviations from this law. One could, e.g., take other combinations of sample log-moments, or simple moments, than the combination used in equation (4). As it was pointed out by G.Molchan in [31], these authors tested the exponential hypothesis H_0 for the distribution of earthquake magnitudes (the ideal G-R law) versus a particular alternative hypothesis H_1, namely, the distribution with a quadratic polynomial in the exponent. They used as statistic the following ratio (we use notations of the present paper that differ from notations in the cited paper):

$$TM = n \sum_{k=1}^{n} \log^2(x_k/u) / \Big(\sum_{k=1}^{n} \log(x_k/u) \Big)^2. \tag{25}$$

There is a simple relation between TM and TP:

$$TM - 1 = 2TP / \Big(1/n \sum_{k=1}^{n} \log^2(x_k/u) \Big). \tag{26}$$

The sum in equation (26) in accordance with the law of Large Numbers converges to its corresponding theoretical log-moment, thus, both $TM - 1$ and TP are close to zero under hypothesis H_0 and deviate from zero if H_0 is not valid. In order to compare their efficiency, we have calculated the TM- and TP-statistics normalized to their std for the subduction sample (fig. 7). We see that both normalized statistics show the same peaks, the TP-statistic being somewhat more efficient. It should be noted that the use of the TM-statistic in the paper [31] assumed *exact magnitude values* which was not quite correct since at that time the Harward catalog still did not exist, and only catalogs with discretized magnitudes were in use, which strictly speaking did not allow using the TM-statistic. For discretized magnitudes, one is forced to use statistics of the TED type.

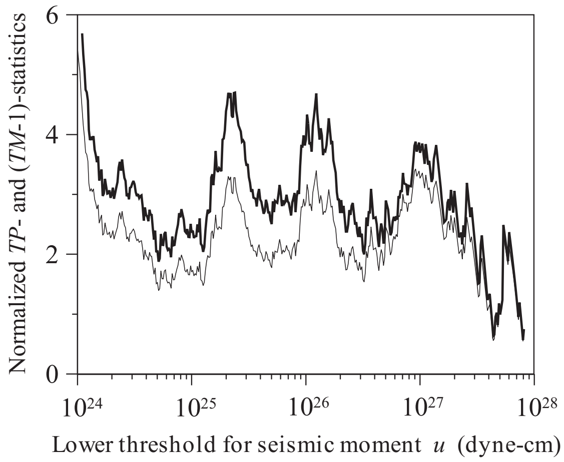


Fig. 7. *TP*-statistic (solid line) and (TM-1)-statistics (thin line) normalized to their std; the subduction sample $n = 4609$

Application of the TP-statistic to subduction zones (the Harvard catalog of seismic moments) permitted to discover some heterogeneities in the seismic moment-frequency curve. They are exhibited more explicitly for subduction zones with low dip angle and with low stress. We cannot say definitely what is the nature of these heterogeneities which appear as oscillations of the TP-statistic in the logarithm of the lower threshold. We can't say that we are absolutely free from some doubts that these discovered oscillations are artifact due to some peculiarities of the algorithms used in the data processing of the Harvard catalog. We can't find neither any definite reason for considering our discovered oscillations as an artifact. So, with these reservations, we shall consider them as a natural effect. As an explanation of these log-periodic oscillations, we can accept that they are the signature of a discrete scale invariance (DSI) in the seismic catalogs. We suggested a simple mechanism in terms of competing faults localized in the domain where the bending of the subducting plate is concentrated.

More generally, if one agrees that the observed log-periodicity is not an artifact and has a natural cause, we would like to emphasize that log-periodicity is an inherent property of systems *with discrete self-similarity*, see [17, 21, 25, 32]. Thus, we can argue that some physical field(s) with discrete self-similarity underlies these oscillation effects. These ideas are close to those suggested by M. Sadovskii in explaining numerous examples of "preferable sizes" in nature: geological blocks, ground particles, dimensions of rock pieces produced by explosions, celestial bodies etc., see [20]. In all these examples, one observes some "humps" of preferred sizes on a smooth background of the distributions. According to Sadovskii, the mean log-distance between two neighboring humps (for linear dimensions of objects, or reduced to linear dimensions) varies from $\log_{10} 2 = 0.3$ to $\log_{10} 4.5 = 0.65$

with average value $\log_{10} 3 \cong 0.5$. If we take our "humps" on the TP-curves for subduction zones and transform their values into linear dimensions (assuming that earthquake energy is proportional to the cubic linear dimension of the source), we get a log-distance about 0.33 (preferred scaling ratio close to 2 for length scales) which falls into the interval indicated by Sadovskii.

Catalogs with discrete magnitudes (quantified with 0.1 of magnitude units) did not show noticeable oscillations in the TED-statistic. A possible reason lies in less precise measurements of earthquake sizes by magnitudes as compared with seismic moments, and in the non-stationarity of catalog time-series. Thus, in our opinion, for detailed statistical analysis of the type performed here, the Harvard catalog of seismic moments is preferable. On the whole, the agreement of regional catalogs of large sizes, say, 10^4 or more, with the G-R law is satisfactory in all ranges, except at the extreme end which needs a special study.

We leave for the future the use of our T-statistic for studies of seismic zonal particularities. This can only be performed under the condition that the corresponding zonal catalog is sufficiently large (perhaps, no less than 10^3 events, depending on the range in question). For zonal catalogs, deviations from the G-R low (if one believes in the zonal validity of the G-R law, and majority of seismologists do believe in such validity) can be connected with tectonic and geological particularities of the zone in question (see for instance [33], which is impossible for global catalogs, or catalogs including several tectonically different areas.

We are grateful to A.Lander for useful discussion and for help in preparing the earthquake catalogs, and to G. Molchan for valuable remarks.

This work was supported by the Russian Fondation of Basic Research, grant 02-05-64379 (Pisarenko, Rodkin). This work is partially supported by NSF-EAR02-30429, by the Southern California Earthquake Center (SCEC) and by the James S. Mc Donnell Foundation 21st century scientist award/ studying complex system. SCEC is funded by NSF Cooperative Agreement EAR-0106924 and USGS Cooperative Agreement 02HQAG0008. The SCEC contribution number for this paper is 751.

REFERENCES

1. *Gutenberg B., Richter C.F.* Seismicity of the Earth and associated phenomena. Princeton Univ. Press: Princeton N.Y., 1954. 310 p.
2. *Pacheco J.F., Sykes L.* Seismic moment catalog of large, shallow earthquakes, 1900-1989 // Bull. Seismol. Soc. Amer. 1992. Vol.82. P.1306–1349.
3. *Okal E.A., Romanowicz B.A.* On the variation of b-values with earthquake size // Phys. Earth Planet. Inter. 1994. Vol.87. P.55–76.

4. *Pisarenko V.F., Sornette D.* Characterization of the frequency of extreme events by the generalized Pareto distribution // PAGEOPH. 2003. Vol.160, N 12. P.2343–2364.
5. *Main I., Burton P.W.* Information theory and the earthquakes frequency-magnitude distribution // Bull. Seismol. Soc. Amer. 1984. Vol.74. P.1409–1426.
6. *Rundle J. B.* Derivation of the complete Gutenberg-Richter magnitude-frequency relation using the principle of scale invariance // J. Geophys. Res. 1989. Vol.94. P.12,337–12,342.
7. *Romanowicz B.* A reappraisal of large earthquake scaling – Comment // Bull. Seismol. Soc. Amer. 1994. Vol.84. P.1765–1776.
8. *Pacheco J.R., Scholz C.H., Sykes L.R.* Changes in frequency-size relationship from small to large earthquakes // Nature. 1992. Vol.335. P.71–73.
9. *Romanowicz B., Rundle J.B.* On scaling relations for large earthquakes // Bull. Seismol. Soc. Amer. 1993. Vol.83. P.1294–1297.
10. *Sornette D., Knopoff L., Kagan Y.Y., Vanneste C.* Rank-ordering statistics of extreme events: application to the distribution of large earthquakes // J. Geophys. Res. 1996. Vol.101. P.13883–13893.
11. *Kagan Y.Y.* Seismic moment-frequency relation for shallow earthquakes: regional comparison // J. Geophys. Res. 1997. Vol.102. P.2835–2852.
12. *Kagan Y.Y.* Universality of seismic moment-frequency relation // PAGEOPH. Vol.155. P.537–573.
13. *Sornette D., Sornette A.* General theory of the modified Gutenberg-Richter law for large seismic moments // Bull. Seismol. Soc. Amer. 1999. Vol.89. P.1121–1130.
14. *Pisarenko V.F., Sornette D.* Statistical detection and characterization of a deviation from the Gutenberg-Richter distribution above magnitude 8. In press in PAGEOPH (preprint at http://arXiv.org/abs/cond-mat/0201552). 2004.
15. *Mandelbrot B.B.* The fractal geometry of nature. San Francisco: W.H. Freeman, 1982. 452 p.
16. *Dubrulle B., Graner F., Sornette D.* (eds.). Scale invariance and beyond. Berlin: EDP Sciences and Springer, 1997. P.286.
17. *Sornette D.* Discrete scale invariance and complex dimensions // Phys. Reports. 1998. Vol.297. P.239–270.
18. *Ouillon G., Sornette D., Genter A., Castaing C.* The imaginary part of rock jointing // J. Phys. I France. 1996. Vol.6, N 8. P.1127–1139.
19. *Huang Y., Ouillon G., Saleur H., Sornette D.* Spontaneous generation of discrete scale invariance in growth models // Phys. Review E. 1997. Vol.55. P.6433–6447.
20. *Sadovskii M.A.* Geophysics and physics of explosion (selected works). Moscow: Nauka, 1999. 335 p. (in Russian).
21. *Geilikman M.B., Pisarenko V.F.* About the selfsimilarity in geophysical phenomena // Discrete Properties of Geophysical Media, M.: Nauka (Ed. M.Sadovskii), P.109–130 (in Russian).
22. *Sahimi S., Arbabi S.* Scaling laws for fracture of heterogeneous materials and rock // Phys. Rev. Lett. 1996. Vol.77. P.3689–3692.
23. *Johansen A., Sornette D.* Evidence of discrete scale invariance by canonical averaging // Inter. J. Mod. Phys. C. 1998. Vol.9, N 3. P.433–447.
24. *Suteanu C., Zugravescu D., Munteanu F.* Fractal approach of structuring by fragmentation // PAGEOPH. 2000. Vol.157, N 4. P.539–557.
25. *Sornette D., Sammis C.G.* Complex critical exponents from renormalization group theory of earthquakes: implication for earthquake predictions // J.Phys. I France. 1995. Vol.5. P.607–619.
26. *Newman W.I., Turcotte D.L., Gabrielov A.M.* Log-periodic behavior of a hierarchical failure model with applications to precursory seismic activation // Phys. Rev. E. 1995. Vol.52. P.4827–4835.

27. *Saleur H., Sammis C.G., Sornette D.* Discrete scale invariance, complex fractal dimensions and log-periodic corrections in earthquakes // J. Geophys. Res. 1996. Vol.101 P.17661–17677.
28. *Rao C.R.* Linear statistical inference and its applications. N-Y: John Wiley, 1973. 625 p.
29. *Embrechts P., Kluppelberg C.P., Mikosh T.* Modelling extremal events. Berlin: Springer-Verlag, 1997. 645 p.
30. *Jarrard R.D.* Relations among subduction parameters // Rev. of Geophys. 1986. Vol.24. P.217–284.
31. *Молчан Г.М., Подгаецкая В.М.* Параметры глобальной сейсмичности // Вычислительные и статистические методы интерпретации сейсмических данных. М.: Наука, 1973. С.44–66. (Вычисл. сейсмология; Вып.6).
32. *Sornette D.* Critical phenomena in natural sciences: chaos, fractals, selforganization and disorder: concepts and tools. Berlin: Springer, 2000. 434 p.
33. *Bird P., Kagan Y.Y, Jackson D.D* Plate tectonics and earthquake potential of spreading ridges and oceanic transform faults // Plate boundary zones / Stein S., Freymueller J.T (eds.). Geodynamics Series. 2002. Vol.30. P.203–218.